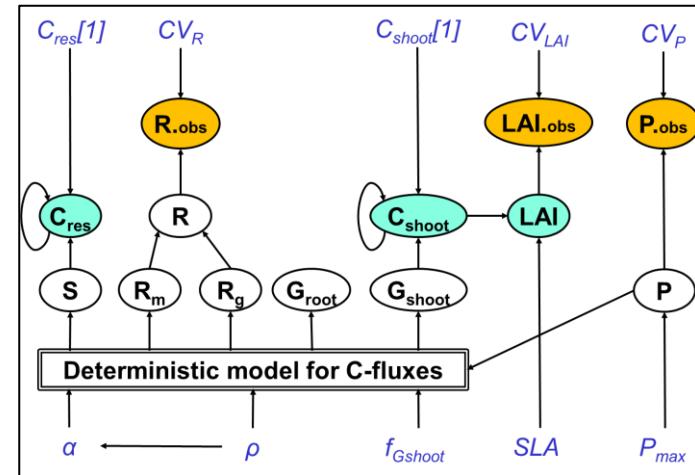
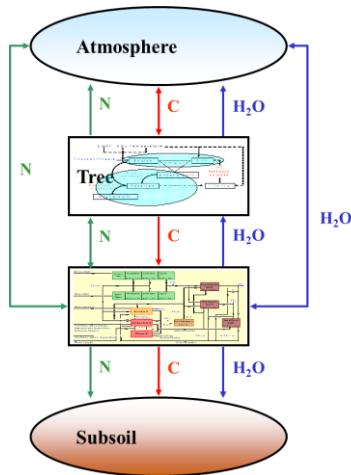


# Bayesian Methods for Ecological and Environmental Modelling: COMMUNICATION



Marcel van Oijen, Lindsay Banin, David Cameron, Peter Levy, Kate Searle

CEH-Edinburgh, 2019-09-12

# Contents

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- 1. Introduction**
- 2. Co-design & graphical modelling**
- 3. Reporting results**
- 4. Visualisation of uncertainty**
- 5. Discussion**

# 1. Introduction

# 1.1 Purpose , ToWhom-What-How, Guidelines?

- **What is the purpose of the communication?**
- **Are you co-designing a model, or is the study finished and are you reporting results?**
  - Graphical modelling is very good for the first purpose, less so for the second
  - Reporting results: *To whom? What? How?*
- **Are there guidelines for visualising uncertainty?**

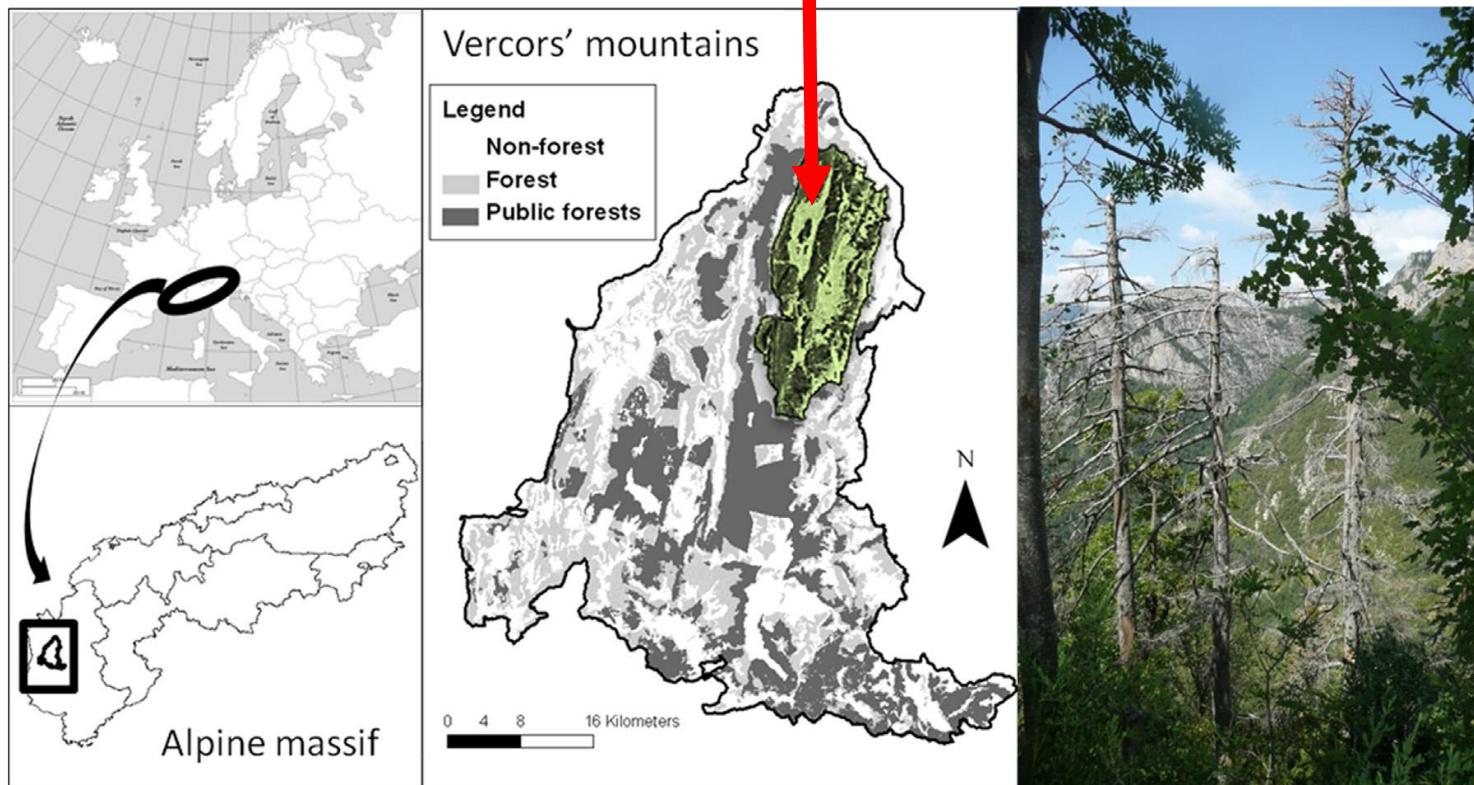
## **2. Co-design & graphical modelling**

## 2.1 Decision-support

***How to make LS-modelling useful for decision-making?***

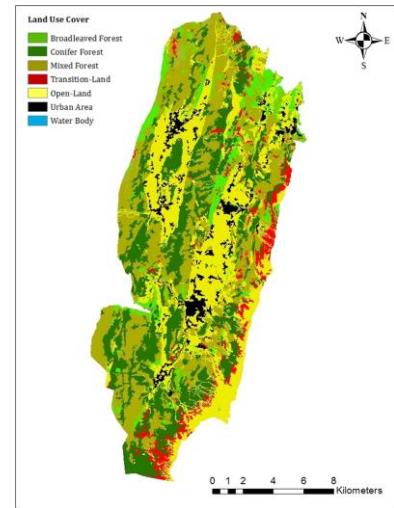
- ***Stimulate involvement of stakeholders***
- ***Recognise uncertainties***

**Production or conservation?**

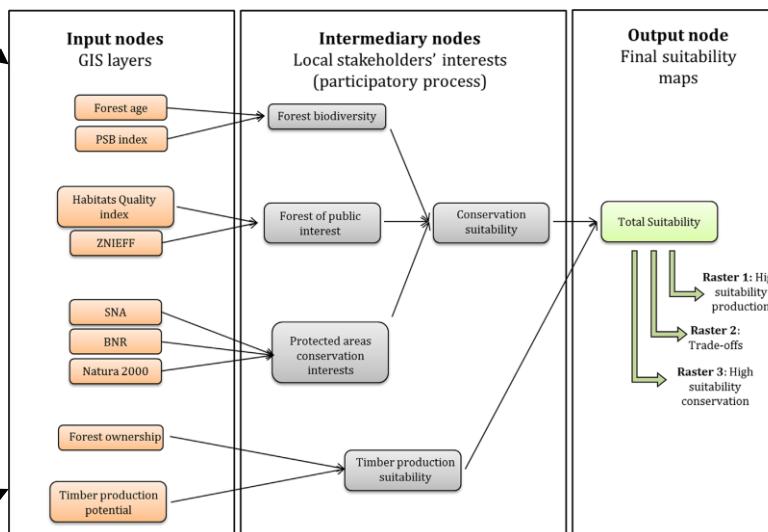


# 2.2 Production or conservation? GIS + BBN

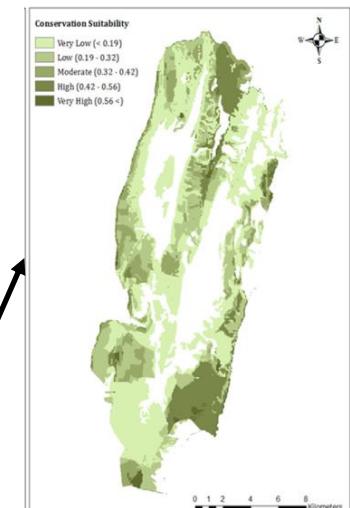
GIS: inputs



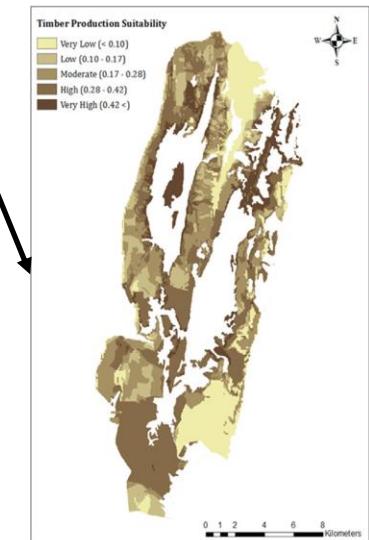
BBN: conditional probabilities



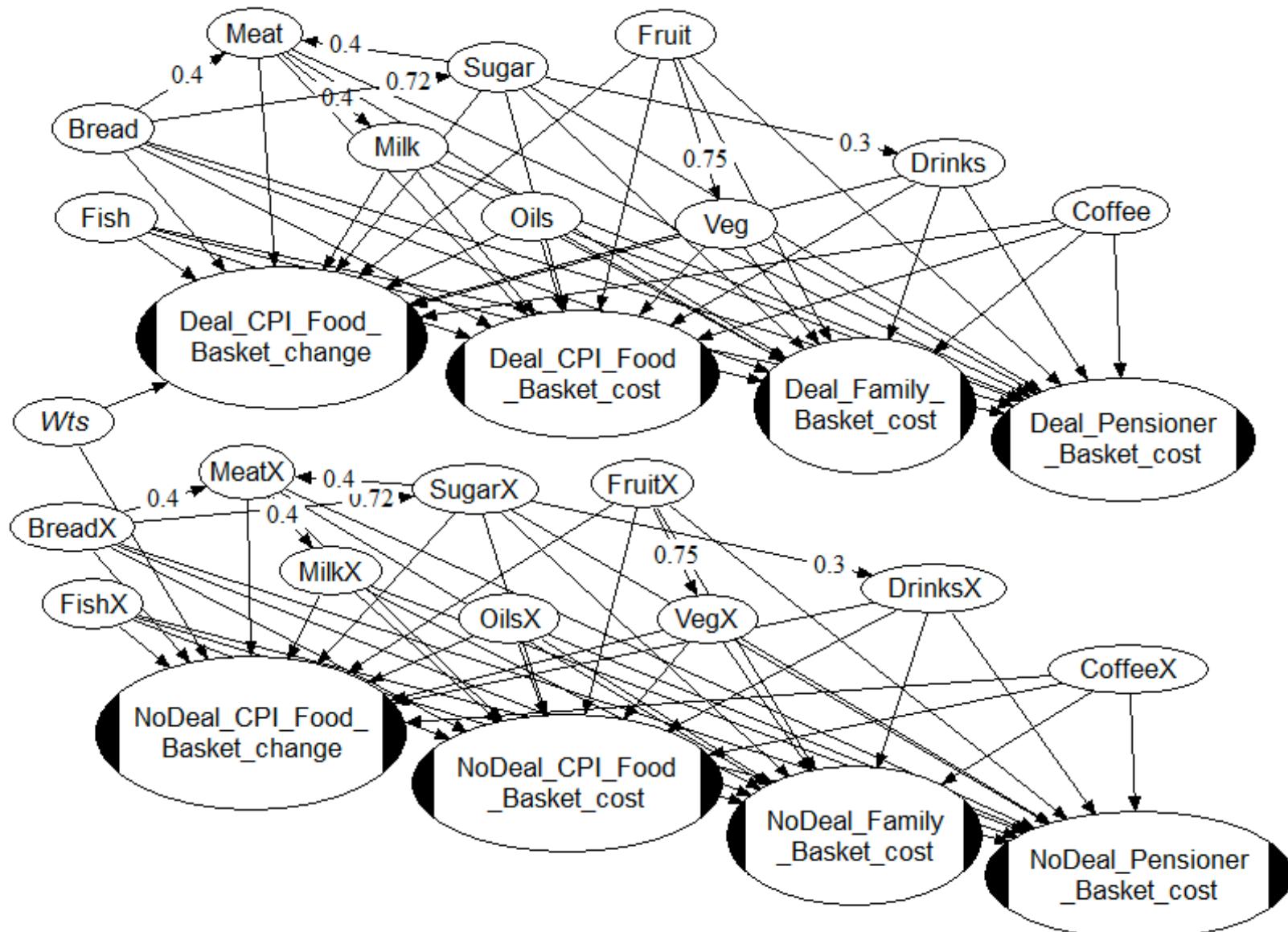
GIS: outputs



Gonzales-Redin et al. (2016)



# Impacts of Brexit scenarios on UK food prices using structured expert judgement



### **3. Reporting results**

# 3.1 Reporting: To whom? What? How

- **Reporting to whom?**
  - Colleagues or stakeholders? Listeners, readers (posters or papers), or app-users?
- **What?**
  - Main message: methodological <-> application?
  - Checklist: Reporting prior, likelihood, algorithm, software, quality test of MCMC, posterior parameter uncertainty, posterior predictive uncertainty
    - *E.g.: beta priors, Gaussian likelihoods for residuals from PBM, Metropolis, BayesianTools, Gelman-Rubin < 1.01, Posterior histograms parameters, Posterior prediction bands for output variables*
- **How?**
  - Text, tables, figures, interactive app

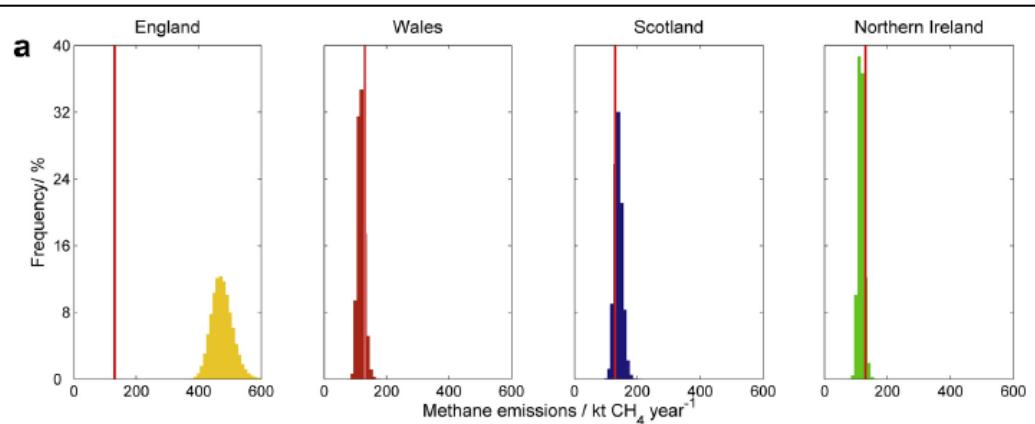
## **4. Visualisation of uncertainty: Guidelines**

# Milne et al. (2015): Communicating the uncertainty in GHG emissions

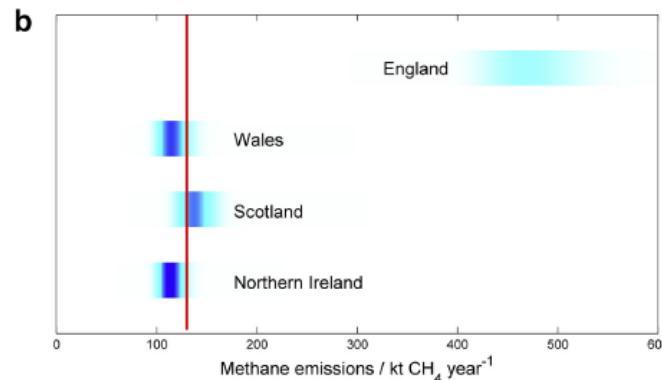
Communicating the uncertainty in estimated greenhouse gas emissions from agriculture

Alice E. Milne <sup>a,\*</sup>, Margaret J. Glendining <sup>a</sup>, R. Murray Lark <sup>b</sup>, Sarah A.M. Perryman  
Taylor Gordon <sup>a</sup>, Andrew P. Whitmore <sup>a</sup> *Journal of Environmental Management* 160 (2015) 139–153

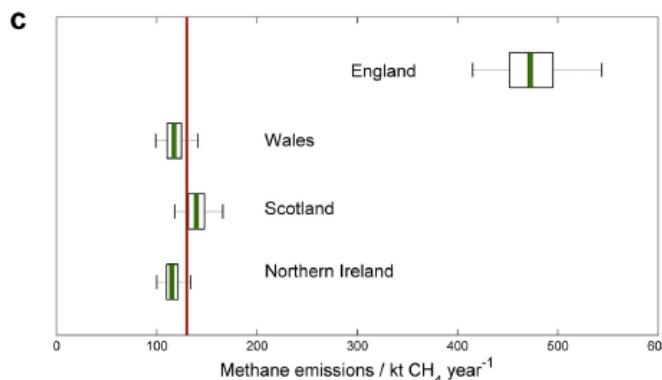
## Histograms



## Shaded arrays

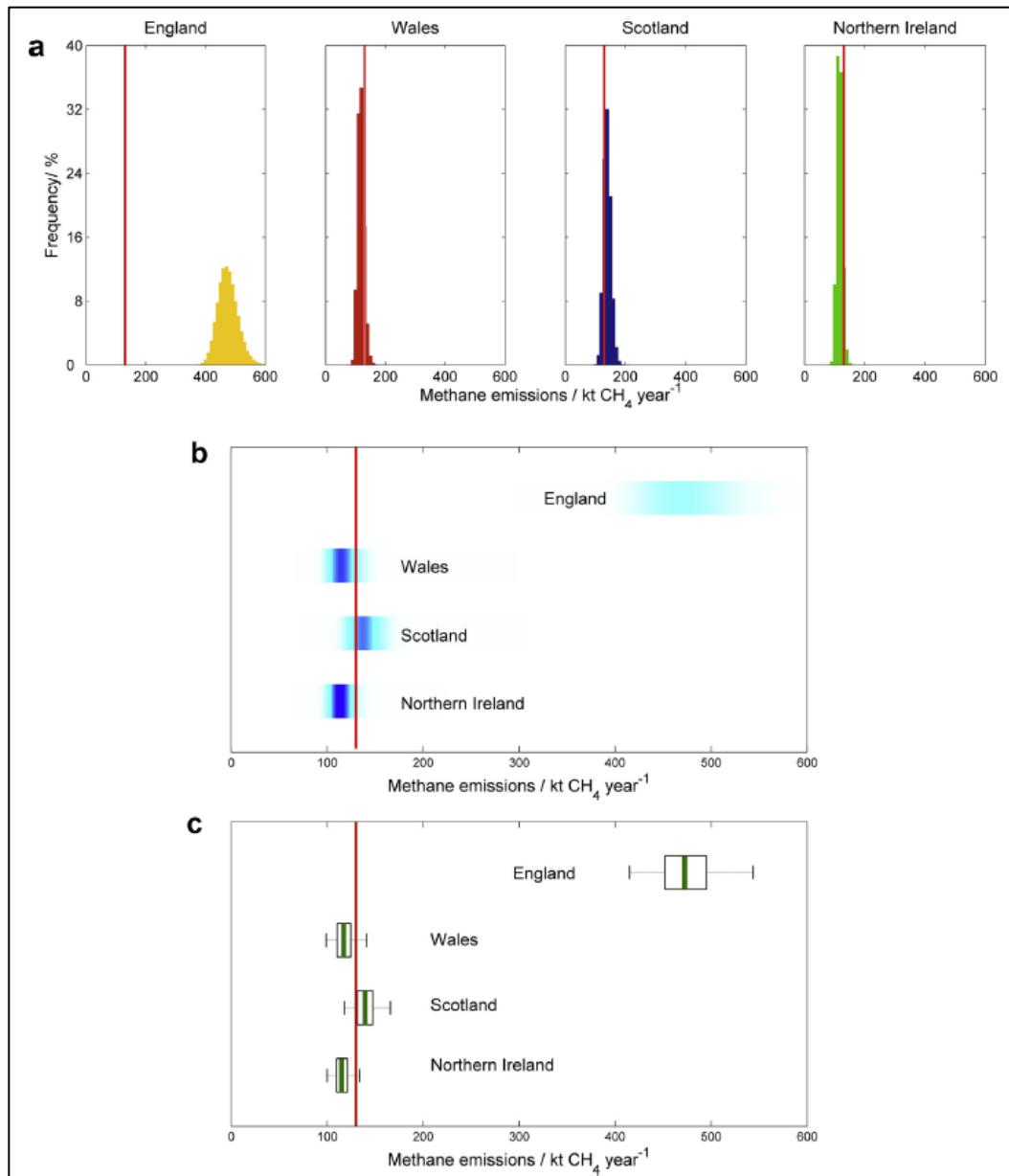


## Boxplots



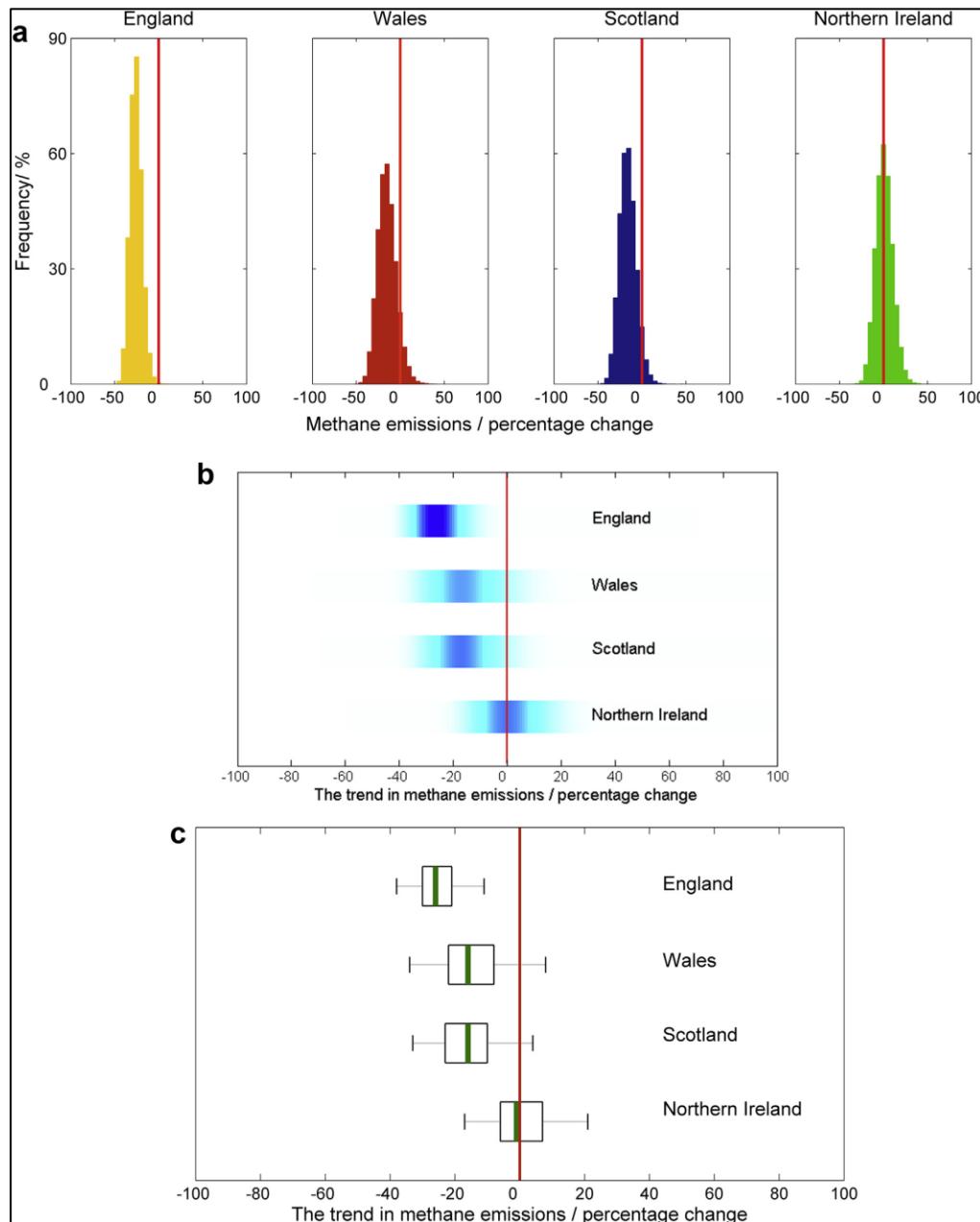
# Milne et al. (2015): Communicating the uncertainty in GHG emissions

■ Is it clear that the estimated emissions are most uncertain for England?

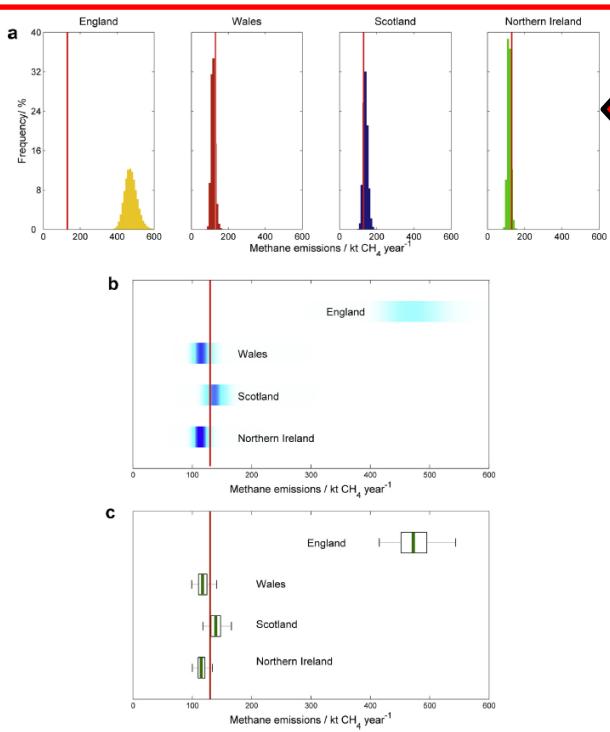


# Milne et al. (2015): Communicating the uncertainty in GHG emissions

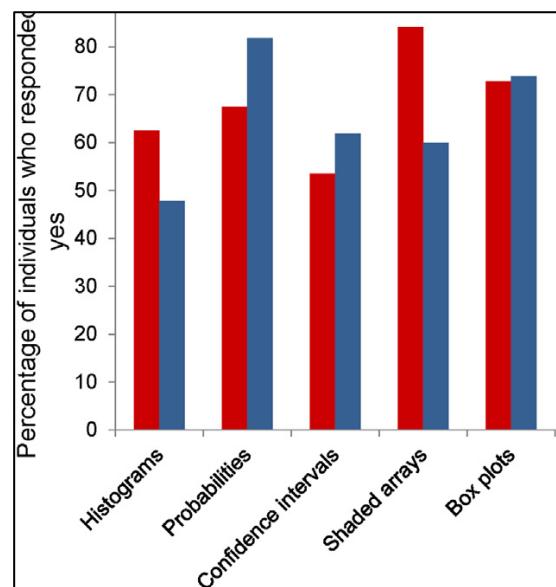
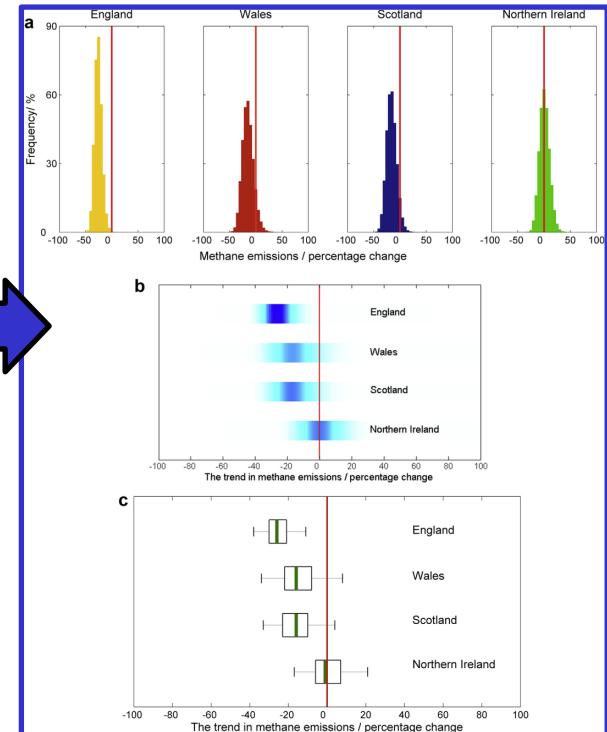
- It is more uncertain that emissions from Scotland have reduced than that emissions from England have reduced?



# Milne et al. (2015): Communicating the uncertainty in GHG emissions

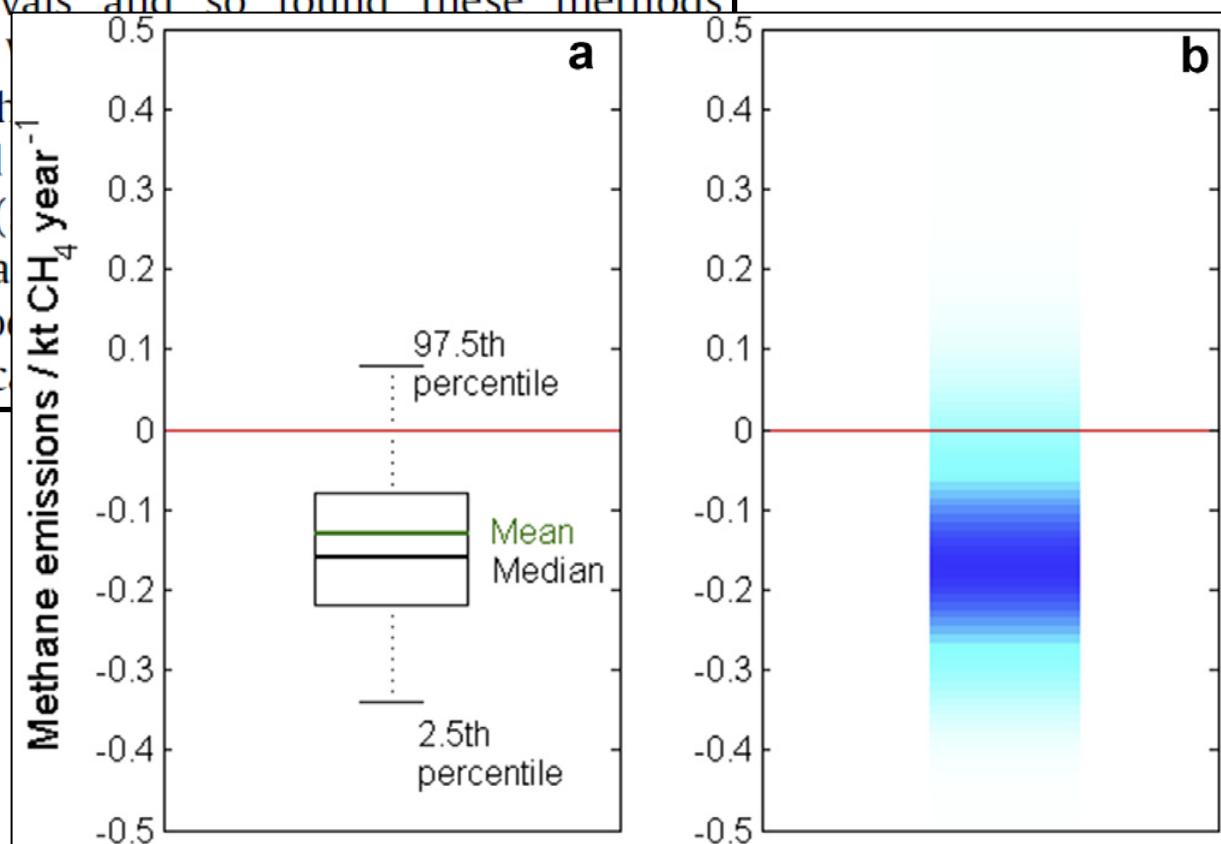


- Is it clear that the estimated emissions are most uncertain for England?
- It is more uncertain that emissions from Scotland have reduced than that emissions from England have reduced?



## 5. Conclusion and recommendation

The methods chosen to communicate uncertainty in estimates of greenhouse gas emissions should be influenced by professional and mathematical background of the target audience. In our study we found that research scientists tended to be familiar with box-plots and confidence intervals and so found these methods straightforward to interpret. with summary statistics such percentiles provide a sound tainty to these individuals ( groups may not be so familiar combination of intuitive methods shaded arrays with numeric



# PRINCIPLES OF POSTERIOR VISUALIZATION (2015)

Blog by Mikhail Shubin:

<https://ctg2pi.wordpress.com/2015/02/24/principles-of-posterior-visualization/>

A) Gamma distribution



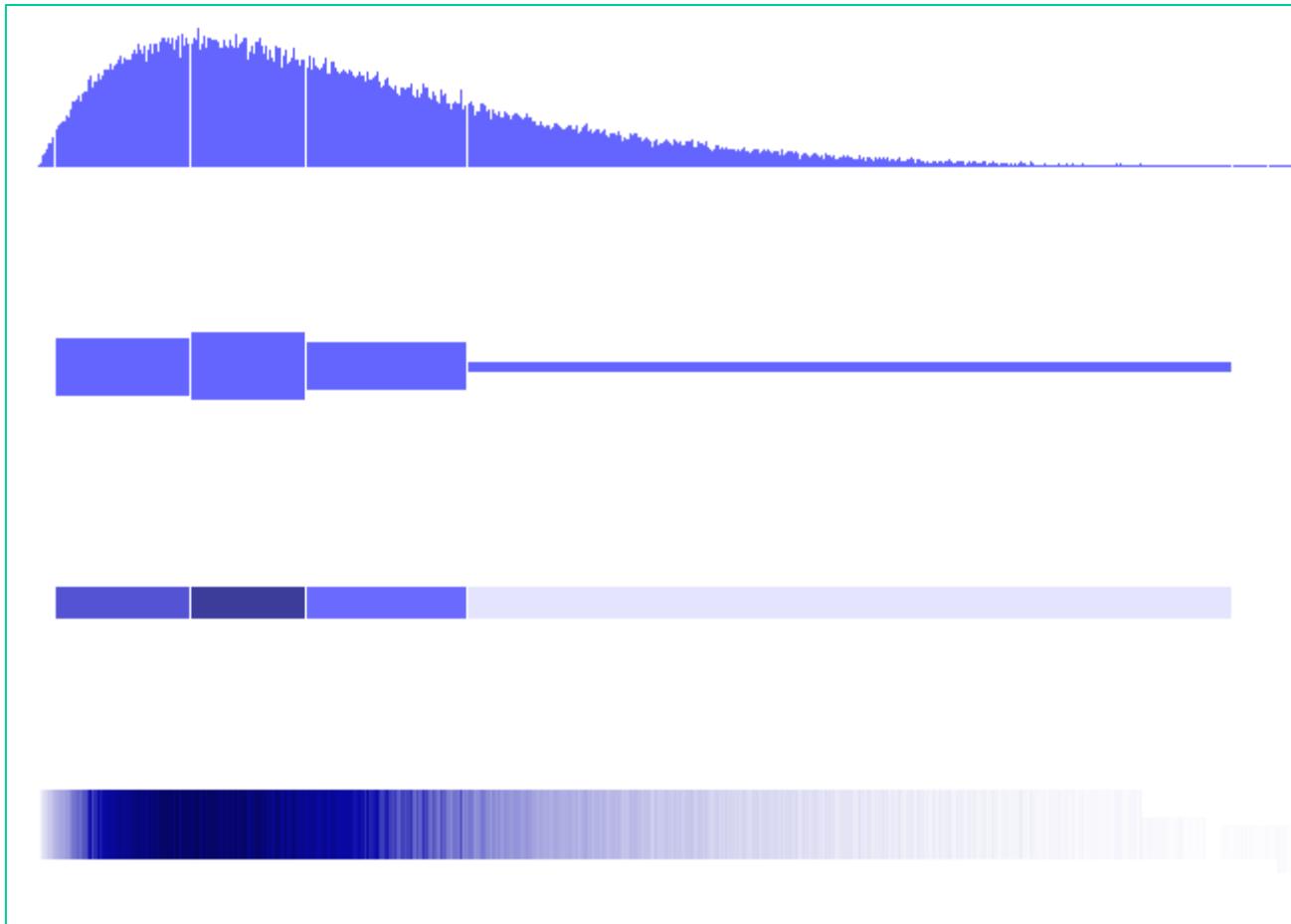
B) Corresponding boxplot.



This and this intervals have almost the same probability to contain the true value (24.5% vs. 25%). However, the second interval looks much bigger.  
Boxplot is misleading

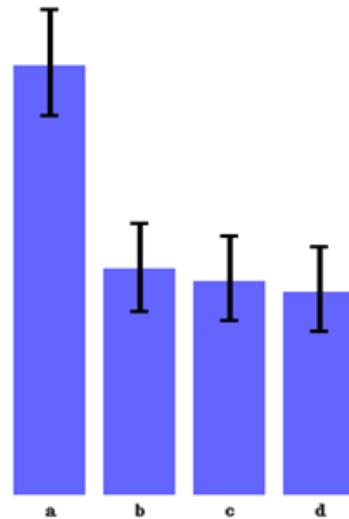
# PRINCIPLES OF POSTERIOR VISUALIZATION (2015)

**Principle: Equal probability = Equal ink**



# PRINCIPLES OF POSTERIOR VISUALIZATION (2015)

## Principle: Do not overemphasize the point estimate



A)

Bars with mean estimates and whiskers representing 90% credible interval.

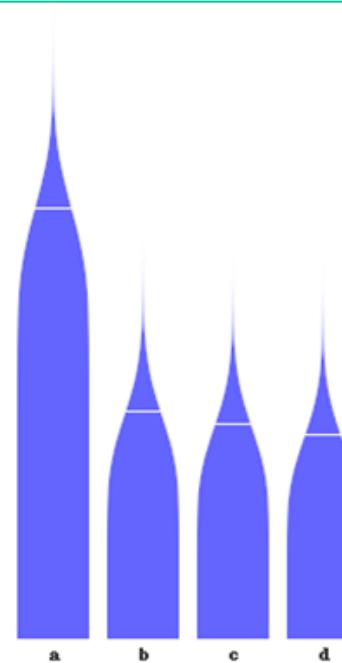
Mean estimates are overemphasized. This figure seems to suggest that the values are ordered:  $a > b > c > d$ . This is true only with probability 0.23



B)

Posterior distributions of the values with mean values highlighted.

Uncertainty in the estimates and the order of values is more visible.

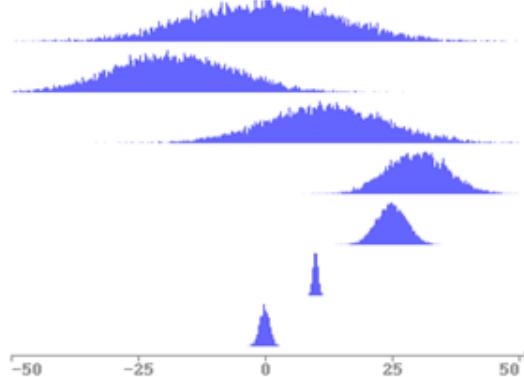


C)

Cumulative distributions of the posterior with mean values highlighted.

The same effect as in (B), but this figure is more similar to the original bars.

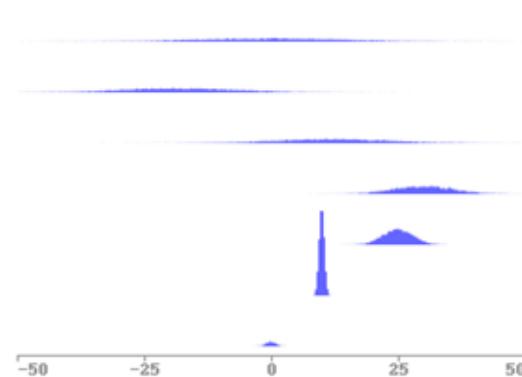
# PRINCIPLES OF POSTERIOR VISUALIZATION (2015)



A)

All histograms have the same height.

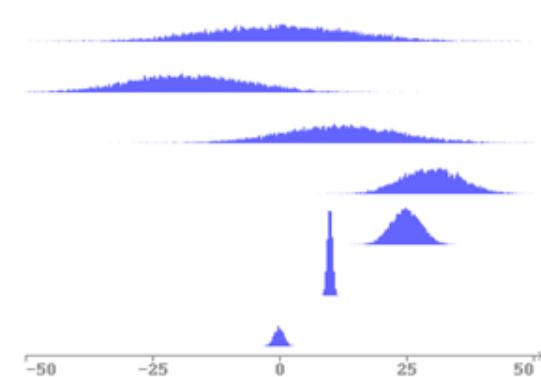
Histograms of the most uncertain estimates dominate the visual space.



B)

Y-axes of all histograms have the same scale.

Histograms of the uncertain estimates are barely readable.

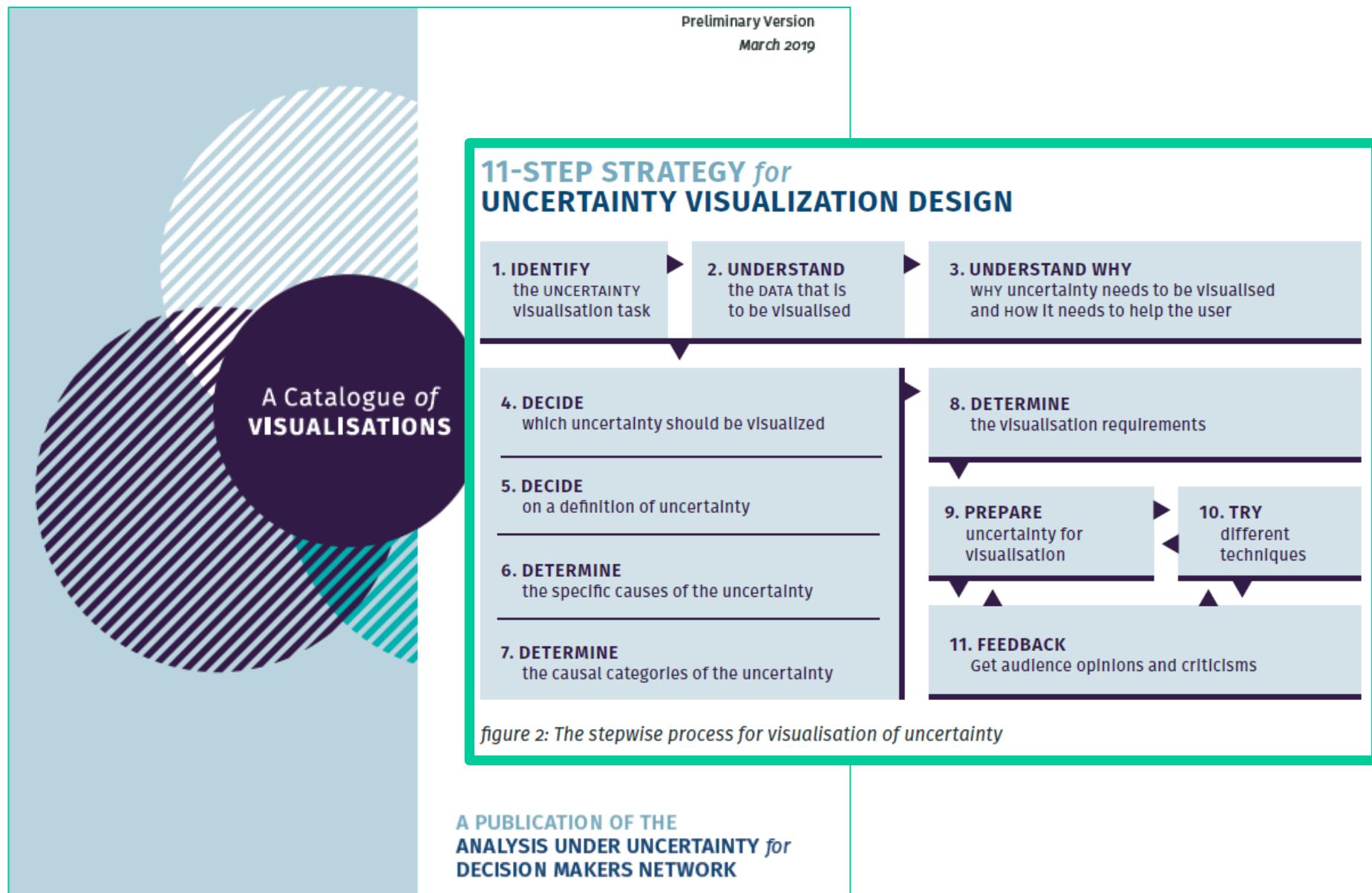


C)

The height of each histogram is proportional to the square root of its height as on Panel B.

Precise estimates are highlighted while uncertain estimates remain readable.

# Kleineberg et al. (2019): Visualizing Uncertainty



# Kleineberg et al. (2019): Visualizing Uncertainty

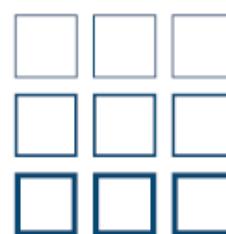
## ATTRIBUTES of GRAPHICS



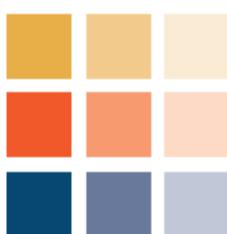
SHAPE



ENCLOSURE



LINE WIDTH



SATURATION



COLOUR HUE



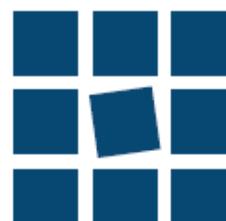
VALUE



SIZE



TEXTURE



ORIENTATION



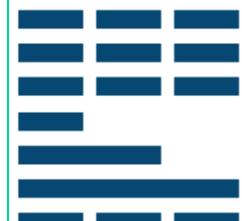
POSITION



3D



JUXTAPOSITION



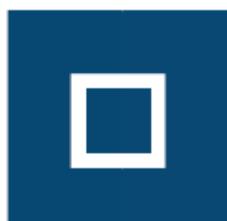
LENGTH



CURVATURE



DENSITY



CLOSURE

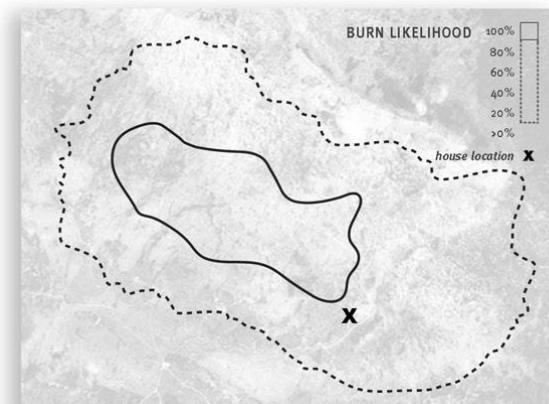


SHARPNESS

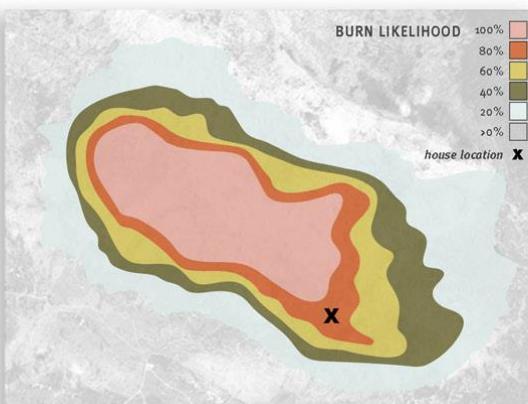


TRANSPARENCY

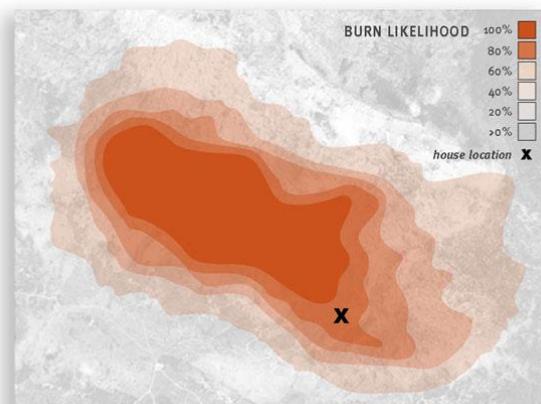
# Kleineberg et al. (2019): Visualizing Uncertainty



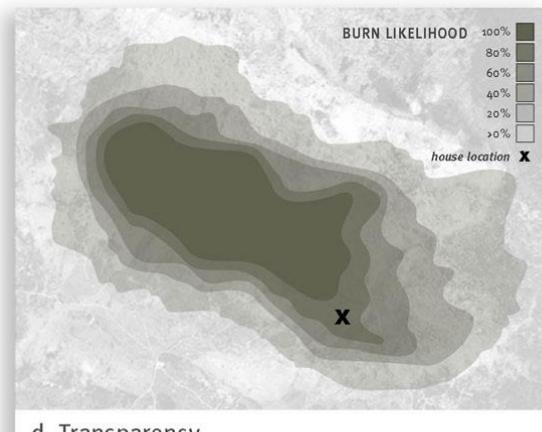
a. Boundary



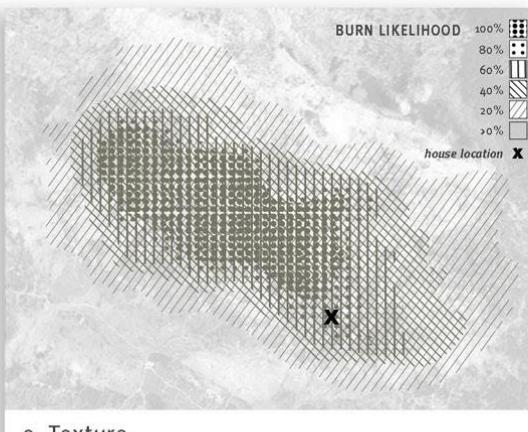
b. Colour hue



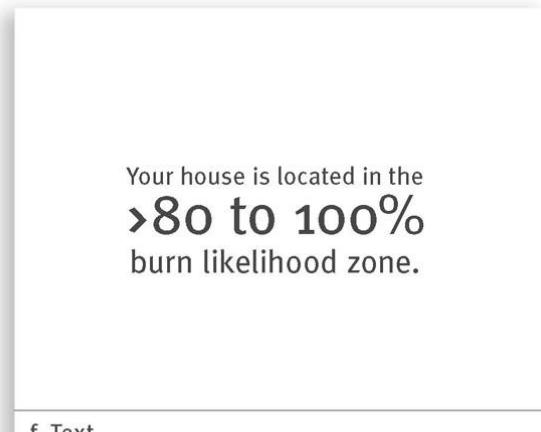
c. Colour value



d. Transparency



e. Texture



f. Text

figure 8: Comparing changing different attributes to communicate uncertainty, research shows that representations involving hue(b), value(c) and transparency(d) work best.

# Kleineberg et al. (2019): Visualizing Uncertainty

Bar chart with error bars

length of bar = proportional to the values they represent.  
whiskers represent error margin (here  $\pm 5$ )

## Resources General Background:

### How do you create good designs?

Nesta Sparks led

<https://www.youcandoit.org/>

### Design

### Visualising things

<https://flowingdata.com/>

### Visualising complex data

figure 13: C <https://www.aclweb.org/anthology/C19-1001.pdf>

### Free images:

<https://pixabay.com/>

<https://unsplash.com/>

<https://thenounproject.com/>

### Catalogue of tools

<http://www.rethinkdb.com/>

<https://datavizcatalogue.com/>

### Tutorials

<https://flowingdata.com/>

## Platforms for Data Visualisations

### Microsoft

<https://powerbi.microsoft.com/en-us/>

### R shiny

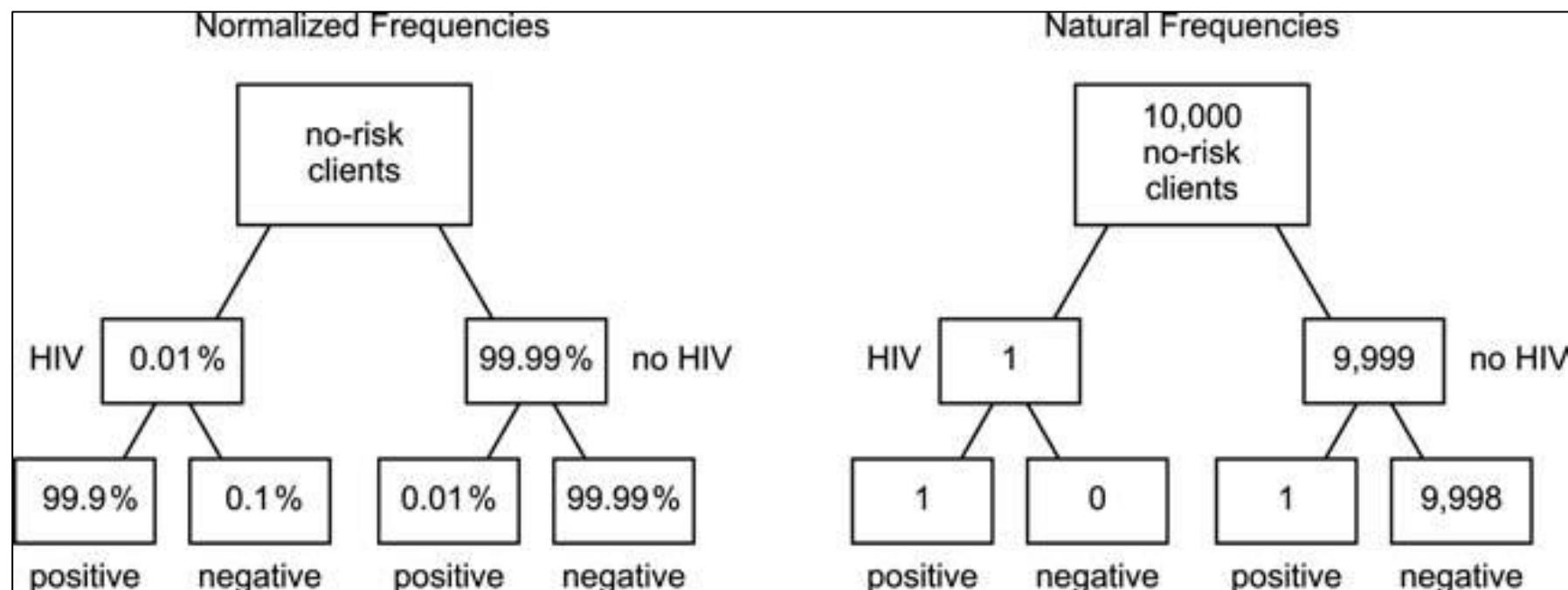
<https://shiny.rstudio.com/gallery/>

### Tableau

<https://www.tableau.com/>

### D3

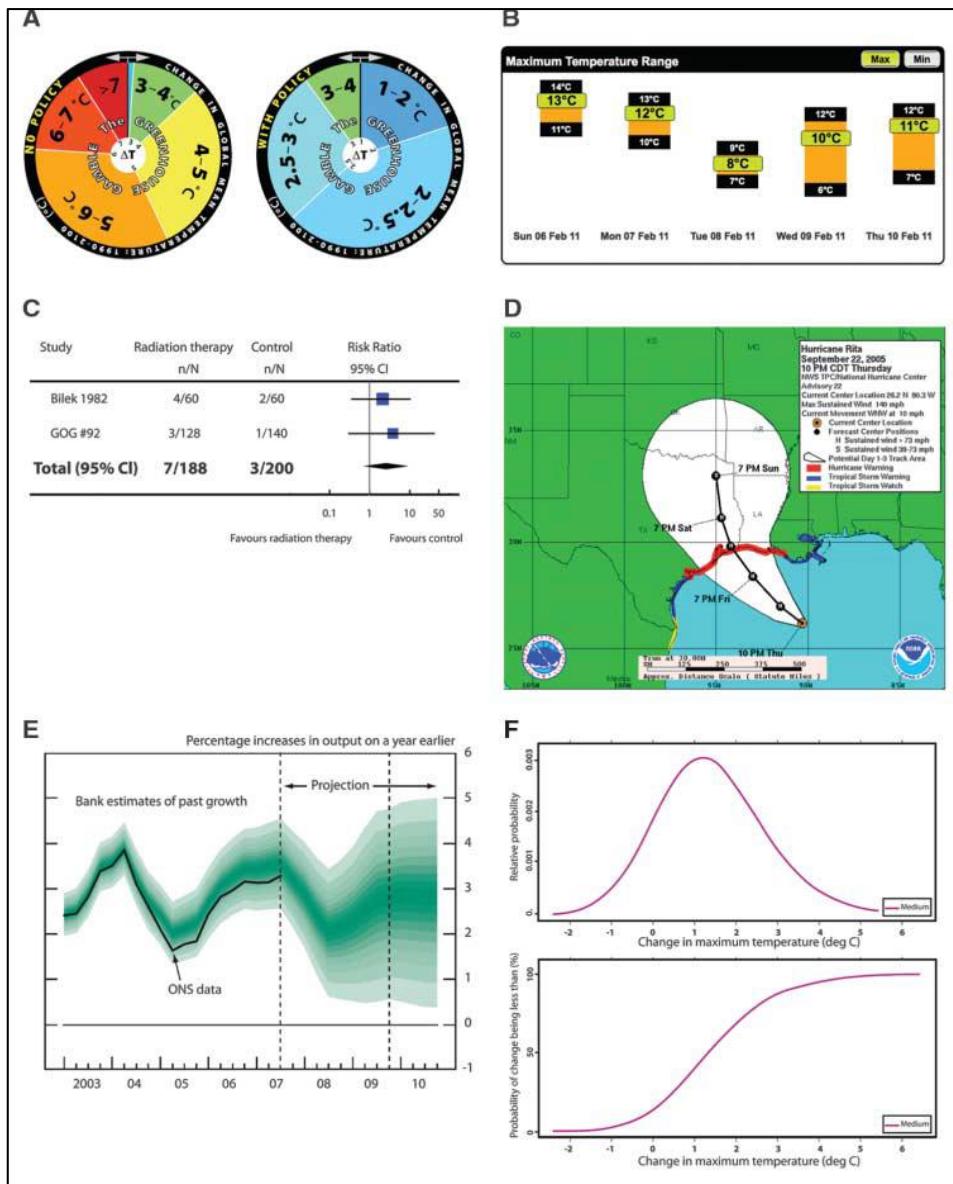
<https://github.com/d3/d3/wiki/Gallery>



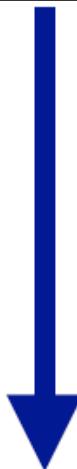
**FIGURE 1.** Two kinds of frequency trees for HIV testing: relative frequencies (left), which are nontransparent for many people, and natural frequencies (right), which are transparent.

# Spiegelhalter et al. (2011): Visualizing uncertainty about the future

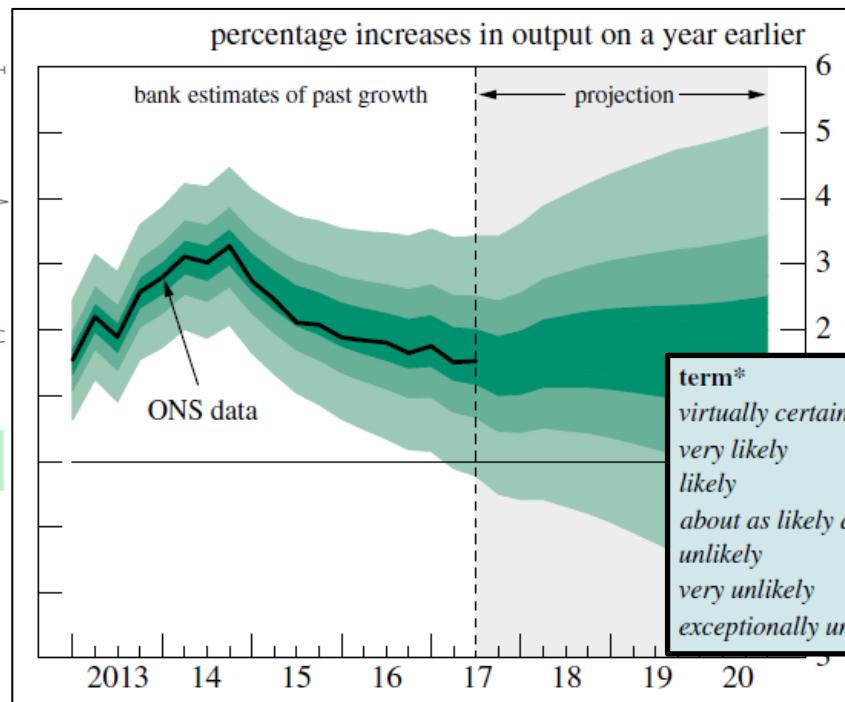
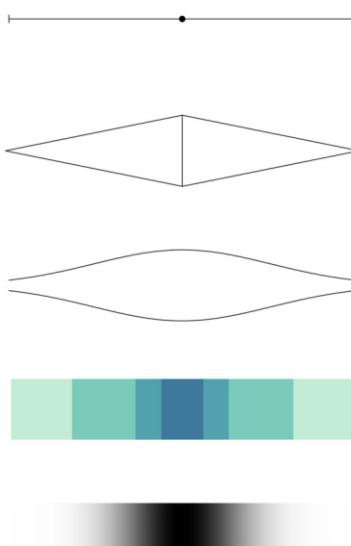
**Fig. 5.** Visualizations of probability distributions for continuous quantities.



Decreasing precision



- i. A full explicit probability distribution
- ii. A summary of a distribution
- iii. A rounded number, range or an order-of-magnitude assessment
- iv. A predefined categorisation of uncertainty
- v. A qualifying verbal statement
- vi. A list of possibilities or scenarios
- vii. Informally mentioning the existence of uncertainty
- viii. No mention of uncertainty
- ix. Explicit denial that uncertainty exists



R. Soc. open sci. 6: 181870.  
<http://dx.doi.org/10.1098/rsos.181870>

## Visualization in Bayesian workflow

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Daniel Simpson‡

*Department of Statistical Sciences, University of Toronto, Canada.*

Aki Vehtari

*Department of Computer Science, Aalto University, Espoo, Finland.*

Michael Betancourt

*ISERP, Columbia University, and Symplectomorphic, LLC, New York, USA.*

Andrew Gelman

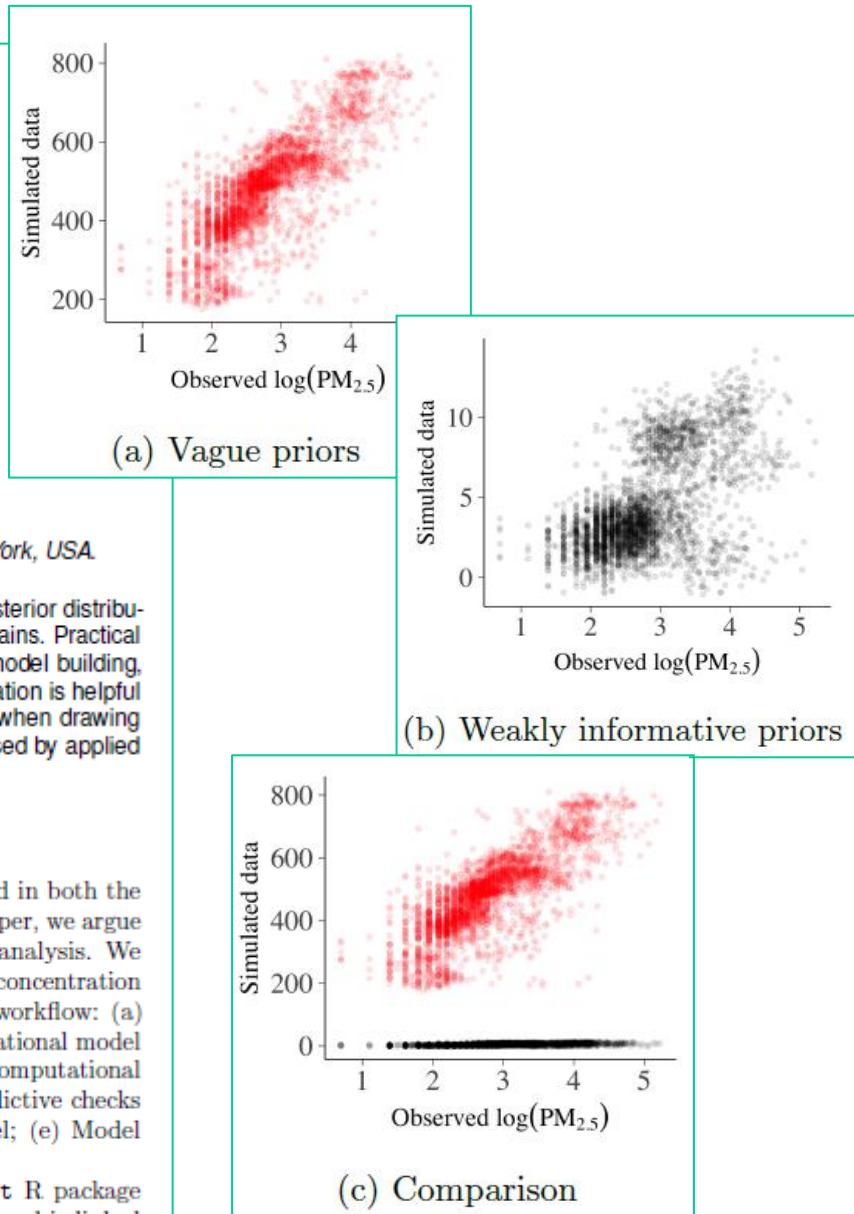
*Departments of Statistics and Political Science, Columbia University, New York, USA.*

**Summary.** Bayesian data analysis is about more than just computing a posterior distribution, and Bayesian visualization is about more than trace plots of Markov chains. Practical Bayesian data analysis, like all data analysis, is an iterative process of model building, inference, model checking and evaluation, and model expansion. Visualization is helpful in each of these stages of the Bayesian workflow and it is indispensable when drawing inferences from the types of modern, high-dimensional models that are used by applied researchers.

### 1. Introduction and running example

Visualization is a vital tool for data analysis, and its role is well established in both the exploratory and final presentation stages of a statistical workflow. In this paper, we argue that the same visualization tools should be used at all points during an analysis. We illustrate this thesis by following a single real example, estimating the global concentration of a certain type of air pollution, through all of the phases of statistical workflow: (a) Exploratory data analysis to aid in setting up an initial model; (b) Computational model checks using fake-data simulation and the prior predictive distribution; (c) Computational checks to ensure the inference algorithm works reliably, (d) Posterior predictive checks and other juxtapositions of data and predictions under the fitted model; (e) Model comparison via tools such as cross-validation.

The tools developed in this paper are implemented in the `bayesplot` R package ([Cran](https://cran.r-project.org/package=bayesplot), 2017, R Core Team, 2017), which uses `gridExtra` (Wickham, 2009) and is linked



# The R Graph Gallery: <https://www.r-graph-gallery.com/>

The R Graph Gallery – Help and info +

CHART TYPES    QUICK    TOOLS    ALL    D3.JS    PYTHON    DATA TO VIZ    ABOUT

Distribution

Violin    Density    Histogram    Boxplot    Ridgeline

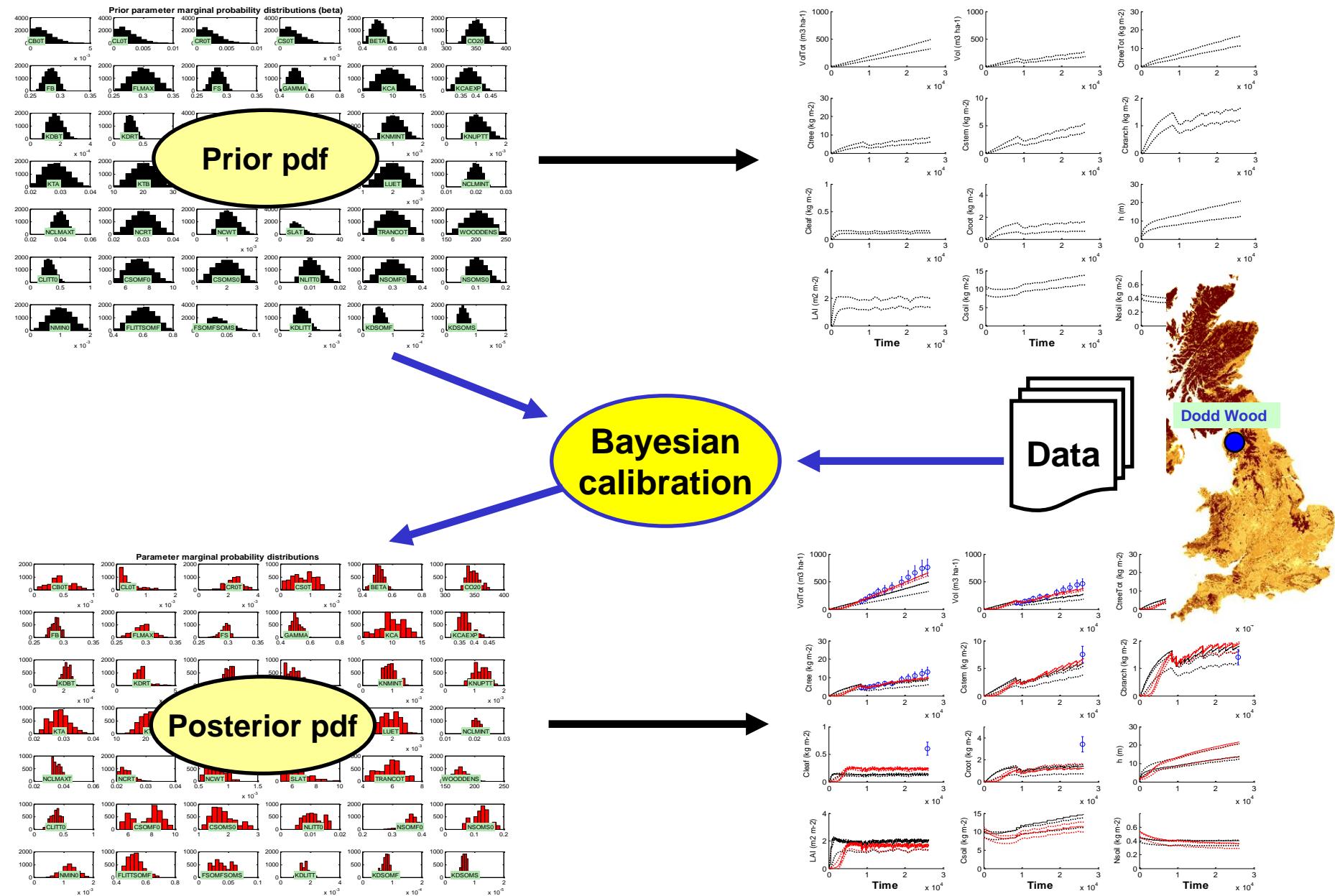
Correlation

Scatter    Heatmap    Correlogram    Bubble    Connected scatter    Density 2d

The screenshot shows the homepage of the R Graph Gallery. At the top, there's a navigation bar with links for CHART TYPES, QUICK, TOOLS, ALL, D3.JS, PYTHON, DATA TO VIZ, and ABOUT. Below this is a search bar and a logo of a heart rate monitor. The main content area is divided into two sections: 'Distribution' and 'Correlation'. The 'Distribution' section contains five icons: Violin (a violin shape), Density (a bell curve), Histogram (bars), Boxplot (a box with whiskers), and Ridgeline (wavy lines). The 'Correlation' section contains six icons: Scatter (dots), Heatmap (grid), Correlogram (matrix of dots), Bubble (dots of varying sizes), Connected scatter (dots connected by lines), and Density 2d (heat map). Each icon has a small caption below it.

## **4. Visualisation of uncertainty: Examples CEH**

# 3.8 Using data in Bayesian calibration of BASFOR



# 3.11 Parameter correlations

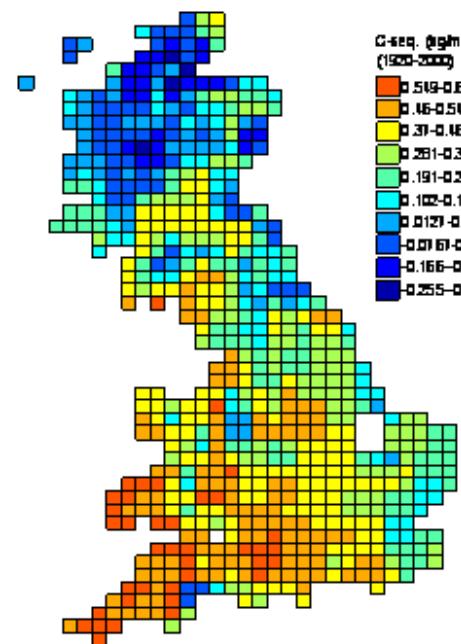
39 parameters

39 parameters

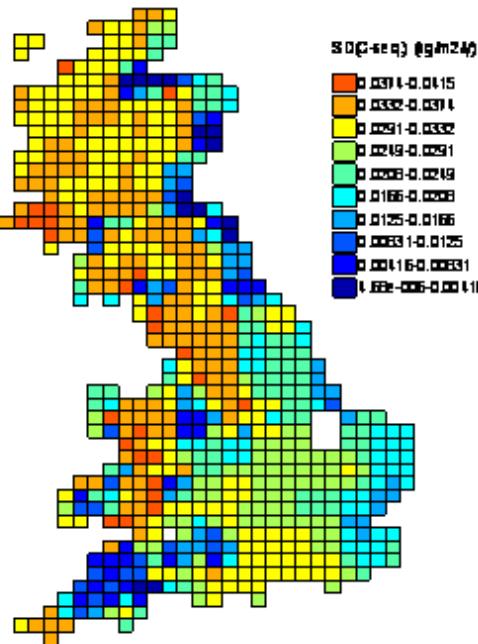
	CL0	CR0	CW0	BETA	CC20	FLMAX	FW	GAMMA	KCA	KCAEXP	KDL	KDR	KDW	KH	KHEXP	KLAIMAX	KMIN	KNUPT	KTA	KTB	KTREE	LUE0	NLCONMIN	NLCONMAX	NRCON	NWCON	SLA	CLITTO	CSOMF0	CSOMS0	NLTTO	NSOMF0	NSOMS0	
CL0	1.00	0.60	-0.67	-0.58	0.25	-0.16	0.51	0.46	0.26	0.12	0.64	0.59	0.38	-0.42	-0.07	0.71	-0.28	0.17	-0.64	-0.32	-0.58	0.23	0.55	0.52	0.12	0.50	-0.58	0.10	0.50	-0.66	-0.57	0.55	0.62	
CR0	0.60	1.00	-0.49	-0.54	0.17	0.40	0.01	0.24	0.51	0.56	0.49	0.96	-0.19	-0.09	0.06	0.55	0.07	0.83	-0.60	-0.81	-0.21	-0.17	0.61	0.67	0.20	0.65	-0.54	-0.05	0.33	-0.29	0.05	0.46	0.61	
CW0	-0.67	-0.49	1.00	0.91	0.24	0.45	-0.70	-0.82	-0.23	0.03	-0.74	-0.57	-0.74	0.77	-0.31	-0.98	0.76	-0.10	0.85	0.14	0.78	-0.61	-0.84	-0.91	0.51	-0.81	0.77	-0.30	-0.38	0.84	0.33	-0.88	-0.90	
BETA	-0.58	-0.54	0.91	1.00	0.30	0.42	-0.78	-0.79	-0.46	-0.08	-0.79	-0.61	-0.66	0.81	0.04	-0.95	0.60	-0.32	0.94	0.17	0.61	-0.59	-0.98	-0.95	0.28	-0.94	0.84	0.01	-0.46	0.83	-0.01	-0.94	-0.96	
CO20	0.25	0.17	0.24	0.30	1.00	0.05	-0.26	-0.41	-0.33	-0.28	0.11	0.09	-0.35	0.67	-0.02	-0.21	0.62	0.00	0.37	0.06	-0.22	-0.76	-0.33	-0.37	0.15	-0.19	0.57	-0.33	-0.34	-0.02	-0.28	-0.54	-0.36	
FLMAX	-0.16	0.40	0.45	0.42	0.05	1.00	-0.69	-0.62	0.43	0.82	-0.56	0.25	-0.87	0.54	-0.05	-0.40	0.59	0.64	0.19	-0.81	0.74	-0.49	-0.31	-0.18	0.61	-0.33	0.06	-0.14	0.21	0.75	0.36	-0.35	-0.21	
FW	0.51	0.01	-0.70	-0.78	-0.26	-0.69	1.00	0.61	0.32	-0.18	0.56	0.05	0.86	-0.83	-0.28	0.77	-0.60	-0.16	-0.75	0.26	-0.55	0.76	0.68	0.58	-0.25	0.58	-0.63	-0.17	0.54	-0.77	-0.13	0.72	0.72	
GAMMA	0.46	0.24	-0.82	-0.79	-0.41	-0.62	0.61	1.00	-0.05	-0.28	0.82	0.45	0.78	-0.82	0.19	0.75	-0.81	-0.06	-0.64	0.14	-0.72	0.63	0.80	0.73	-0.46	0.78	-0.65	0.49	0.06	-0.85	-0.31	0.87	0.67	
KCA	0.26	0.51	-0.23	-0.46	-0.33	0.43	0.32	-0.05	1.00	0.84	-0.01	0.38	-0.10	-0.34	-0.49	0.39	0.07	0.72	-0.68	-0.69	0.35	0.30	0.49	0.51	0.47	0.37	-0.69	-0.49	0.86	0.05	0.54	0.45	0.62	
KCAEXP	0.12	0.56	0.03	-0.08	-0.28	0.82	-0.18	-0.28	0.84	1.00	-0.30	0.41	-0.48	0.00	-0.24	0.76	-0.36	-0.91	0.59	0.01	0.16	0.27	0.59	0.06	-0.48	-0.22	0.68	0.42	0.44	0.16	0.32			
KDL	0.64	0.49	-0.74	-0.79	0.11	-0.56	0.56	0.82	-0.01	-0.30	1.00	0.64	0.56	-0.53	-0.03	0.73	-0.39	0.17	-0.61	0.07	-0.81	0.21	0.81	0.67	-0.25	0.88	-0.48	0.10	-0.02	0.93	-0.25	0.70	0.63	
KDR	0.59	0.96	-0.57	-0.61	0.09	0.25	0.05	0.45	0.38	0.41	0.64	1.00	-0.06	-0.20	0.12	0.59	-0.07	0.75	-0.61	-0.69	-0.34	-0.10	0.70	0.72	0.09	0.75	-0.57	0.10	0.19	-0.42	0.01	0.57	0.63	
KDW	0.38	-0.19	-0.74	-0.66	-0.35	-0.87	0.86	0.78	-0.10	-0.48	0.56	-0.06	1.00	-0.84	0.12	0.70	-0.86	-0.49	-0.54	0.49	-0.73	0.81	0.54	0.50	-0.60	0.47	-0.48	0.29	0.21	-0.81	0.41	0.67	0.56	
KH	-0.42	-0.09	0.77	0.81	0.67	0.54	-0.83	-0.82	-0.34	0.00	-0.53	-0.20	-0.84	1.00	0.07	-0.78	0.85	0.08	0.80	-0.07	0.44	-0.93	-0.77	-0.73	0.30	-0.64	0.84	-0.25	-0.52	0.68	0.12	-0.92	-0.79	
KHEXP	-0.07	0.06	-0.31	0.04	-0.02	-0.05	-0.28	0.19	-0.49	-0.24	-0.03	0.12	0.12	0.07	1.00	0.14	-0.43	-0.46	0.26	0.14	0.00	-0.40	-0.01	0.12	0.15	-0.76	-0.05	0.12	0.72	-0.37	-0.05	-0.47	-0.02	0.00
KLAIMAX	0.71	0.55	-0.98	-0.95	-0.21	-0.40	0.77	0.75	0.38	0.07	0.73	0.59	0.70	-0.78	0.14	1.00	-0.67	0.21	-0.93	-0.21	-0.70	0.60	0.88	0.93	-0.38	0.83	-0.82	0.11	0.51	-0.83	-0.22	0.89	0.96	
KMIN	-0.28	0.07	0.76	0.60	0.62	0.59	-0.60	-0.81	0.07	0.24	-0.39	-0.07	-0.86	0.85	-0.43	-0.67	1.00	0.38	0.53	-0.22	0.58	-0.86	-0.52	-0.59	0.66	-0.42	0.60	-0.63	-0.22	0.61	0.42	-0.73	-0.58	
KNUPT	0.17	0.83	-0.10	-0.32	0.00	0.64	-0.16	-0.06	0.72	0.76	0.17	0.75	-0.49	0.08	-0.26	0.21	0.38	1.00	-0.43	-0.83	0.28	-0.27	0.45	0.46	0.47	0.48	-0.41	-0.38	0.33	0.10	0.58	0.26	0.41	
KTA	-0.64	-0.60	0.85	0.94	0.37	0.19	-0.75	-0.64	-0.68	-0.36	-0.61	-0.61	-0.54	0.80	0.14	-0.93	0.53	-0.43	1.00	0.39	0.40	-0.64	-0.92	-0.93	0.08	-0.83	0.94	0.07	-0.71	0.66	-0.05	-0.92	-0.99	
KTB	-0.32	-0.81	0.14	0.17	0.06	-0.81	0.26	0.14	-0.69	-0.91	0.07	-0.69	0.49	-0.07	0.00	-0.21	-0.22	-0.83	0.39	1.00	-0.33	0.16	-0.25	-0.39	-0.46	-0.21	0.47	0.05	-0.52	-0.25	-0.22	-0.21	-0.38	
KTREE	-0.58	-0.21	0.78	0.61	-0.22	0.74	-0.55	-0.72	0.35	0.59	-0.81	-0.34	-0.73	0.44	-0.40	-0.70	0.58	0.28	-0.40	1.00	-0.26	0.52	0.51	0.66	-0.58	0.24	-0.32	0.15	0.91	0.60	-0.50	-0.48		
LUE0	0.23	-0.17	-0.61	-0.59	-0.76	-0.49	0.76	0.63	0.30	0.01	0.21	-0.10	0.81	-0.93	-0.01	0.60	-0.86	-0.27	-0.64	0.16	-0.26	1.00	0.52	0.53	-0.33	0.35	-0.72	0.28	0.56	-0.45	-0.13	0.73	0.62	
NLCONMIN	0.55	0.61	-0.84	-0.98	-0.33	-0.31	0.68	0.80	0.49	0.16	0.81	0.70	0.54	-0.77	-0.12	0.88	-0.52	0.45	-0.92	-0.25	-0.52	0.52	1.00	0.94	-0.16	0.97	-0.85	0.00	0.41	-0.77	0.10	0.95	0.92	
NLCONMAX	0.52	0.67	-0.91	-0.95	-0.37	-0.18	0.58	0.73	0.51	0.27	0.67	0.72	0.50	-0.73	0.15	0.93	-0.59	0.46	-0.93	-0.39	-0.51	0.53	0.94	1.00	-0.32	0.91	-0.87	0.11	0.46	-0.67	0.05	0.92	0.96	
NRCON	0.12	0.20	0.51	0.29	0.15	0.61	-0.25	-0.46	0.47	0.59	-0.25	0.09	-0.60	0.30	-0.76	-0.38	0.66	0.47	0.08	-0.46	0.66	-0.33	-0.16	-0.32	1.00	-0.22	-0.01	-0.46	0.34	0.44	0.31	-0.23	-0.21	
NWCON	0.50	0.65	-0.81	-0.94	-0.19	-0.33	0.58	0.78	0.37	0.06	0.88	0.75	0.47	-0.64	-0.05	0.83	-0.42	0.48	-0.83	-0.21	-0.58	0.35	0.97	0.91	-0.22	1.00	-0.72	-0.03	0.23	-0.79	0.12	0.86	0.85	
SLA	-0.58	-0.54	0.77	0.84	0.57	0.06	-0.63	-0.65	-0.69	-0.48	-0.48	-0.57	0.84	0.12	-0.82	0.60	-0.41	0.94	0.47	0.24	-0.72	-0.85	-0.07	-0.01	-0.72	1.00	-0.13	-0.13	-0.75	0.51	-0.03	-0.93	-0.92	
CLITTO	0.10	-0.05	-0.30	0.01	-0.33	-0.14	-0.17	0.49	-0.49	-0.22	0.10	0.10	0.29	-0.25	0.72	0.11	-0.63	-0.38	0.07	0.05	-0.32	0.28	0.00	0.11	-0.46	-0.03	-0.13	1.00	-0.25	-0.15	-0.64	0.22	0.00	
CSOMF0	0.50	0.33	-0.38	-0.46	-0.34	0.21	0.54	0.06	0.86	0.68	-0.02	0.19	0.21	-0.52	-0.37	0.51	-0.22	0.33	-0.71	-0.52	0.15	0.56	0.41	0.46	0.34	0.23	-0.75	-0.25	1.00	-0.10	0.09	0.50	0.65	
CSOMS0	-0.66	-0.29	0.84	0.83	-0.02	0.75	-0.77	-0.85	0.05	0.42	-0.93	-0.42	-0.81	0.68	-0.05	-0.83	0.61	0.10	0.66	-0.25	0.91	-0.45	-0.77	-0.67	0.44	-0.79	0.51	-0.15	-0.10	1.00	0.39	-0.74	-0.68	
NLTTO	-0.57	0.05	0.33	-0.01	-0.28	0.36	-0.13	-0.31	0.54	0.44	-0.25	-0.01	-0.41	0.12	-0.47	-0.22	0.42	0.58	-0.05	-0.22	0.60	-0.13	0.10	0.05	0.31	0.12	-0.03	-0.64	0.09	0.39	1.00	-0.05	0.01	
NSOMF0	0.55	0.46	-0.88	-0.94	-0.54	-0.35	0.72	0.87	0.45	0.16	0.70	0.57	0.67	-0.92	-0.02	0.89	-0.73	0.26	-0.92	-0.21	-0.50	0.70	0.95	0.92	-0.23	0.86	-0.93	0.22	0.50	-0.74	-0.05	1.00	0.92	
NSOMS0	0.62	0.61	-0.90	-0.96	-0.36	-0.21	0.72	0.67	0.62	0.32	0.63	0.63	0.56	-0.79	0.00	0.96	-0.58	0.41	-0.99	0.62	0.92	0.96	-0.21	0.85	-0.92	0.00	0.65	-0.68	0.01	0.92	1.00			
NMIN0	-0.16	-0.31	-0.47	-0.41	-0.64	-0.43	0.56	0.33	0.16	-0.09	-0.06	-0.30	0.66	-0.63	0.29	0.45	-0.72	-0.33	-0.40	0.25	-0.21	0.79	0.27	0.41	-0.66	0.16	-0.39	0.14	0.33	-0.23	0.06	0.42	0.45	
FLITTSOMF	0.48	0.60	-0.01	0.08	0.61	0.31	-0.43	0.03	-0.22	0.05	0.36	0.63	-0.39	0.40	0.15	-0.02	0.34	0.33	0.12	-0.39	-0.22	-0.62	0.01	-0.02	0.29	0.13	0.12	0.23	-0.28	-0.11	-0.40	-0.10	-0.10	
FSOMFSOMS	-0.66	-0.28	0.86	0.83	0.08	0.55	-0.89	-0.56	-0.33	0.08	-0.58	-0.27	-0.78	0.72	-0.04	-0.91	0.61	0.04	0.81	-0.03	0.69	-0.63	-0.69	-0.72	0.41	-0.62	0.65	0.07	-0.55	0.78	0.27	-0.70	-0.83	
KDLITT	0.42	0.28	-0.93	-0.89	-0.55	-0.51	0.73	0.87	0.25	-0.04	0.62	0.39	0.81	-0.91	0.26	0.90	-0.88	0.02	-0.83	-0.01	-0.63	0.80	0.84	0.88	-0.56	0.77	-0.80	0.34	0.37	-0.75	-0.16	0.92	0.87	
KDSOMF0	0.15	-0.43</																																

### 3.11 Maps of (1) predictions, (2) uncertainties

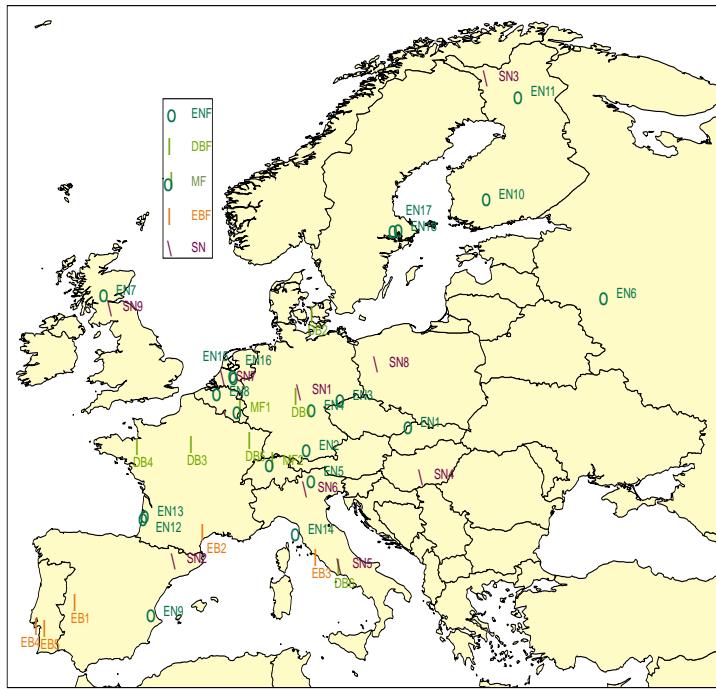
C-sequestration  
(model output for  
1920-2000)



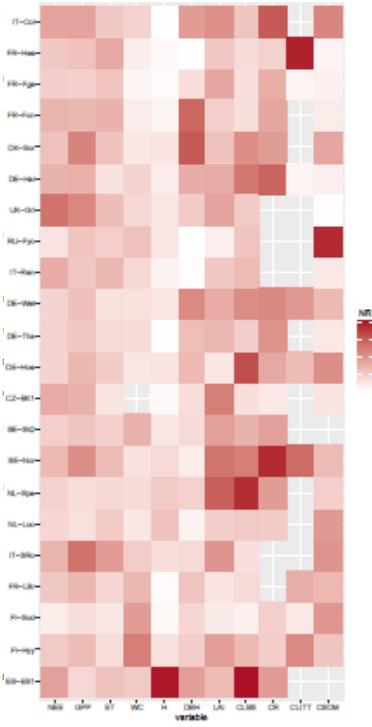
Uncertainty of  
C-sequestration



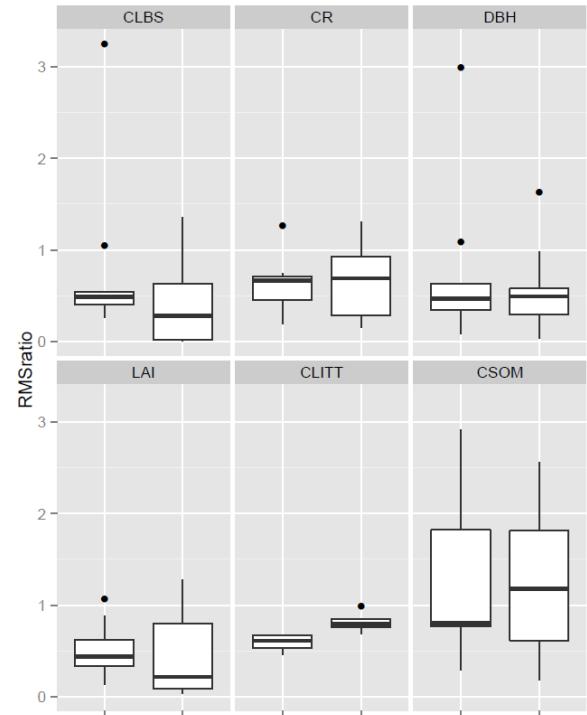
# 4.1 Site-specific vs. generic BC for European forest



*RMS - ratio*



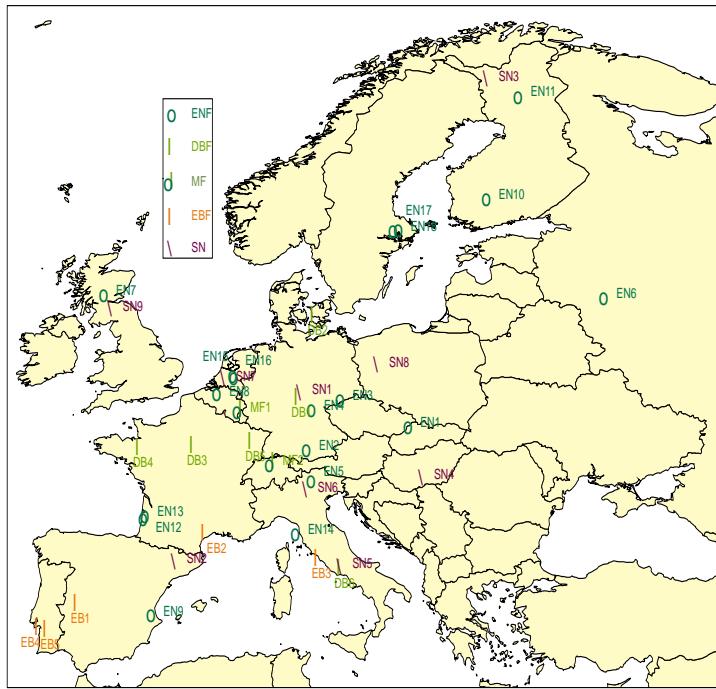
*RMS - ratio*



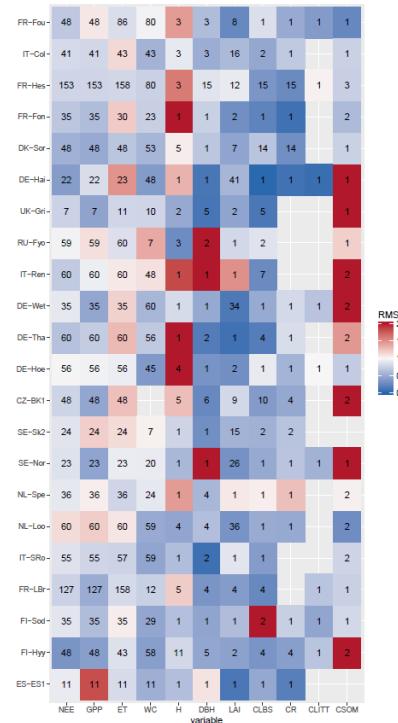
Generic BC

Site-specific BC

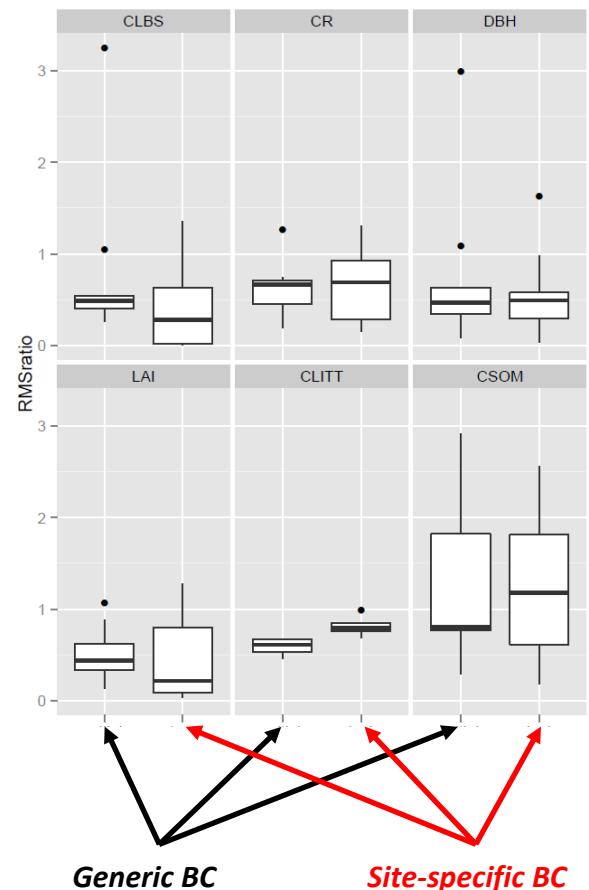
# 4.1 Site-specific vs. generic BC for European forest



*RMS - ratio*

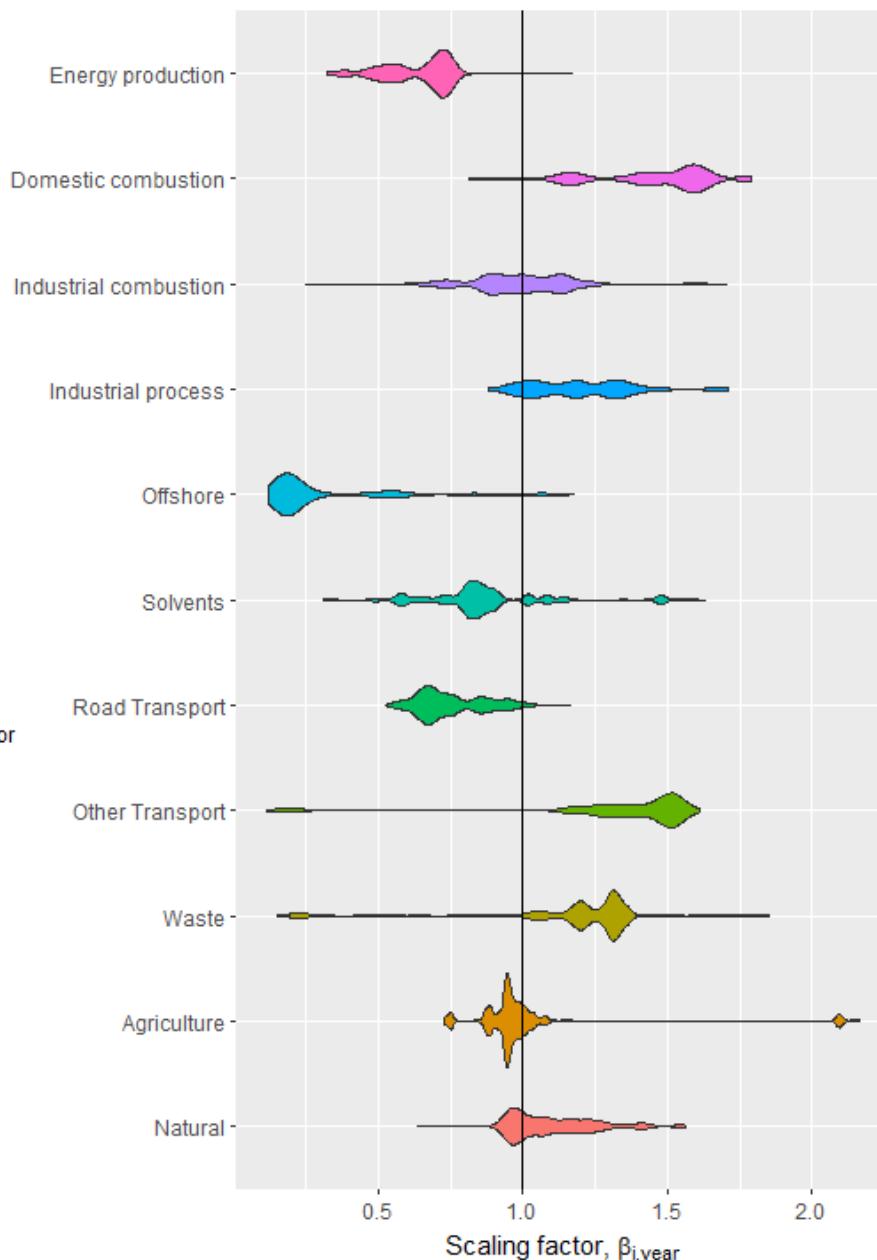
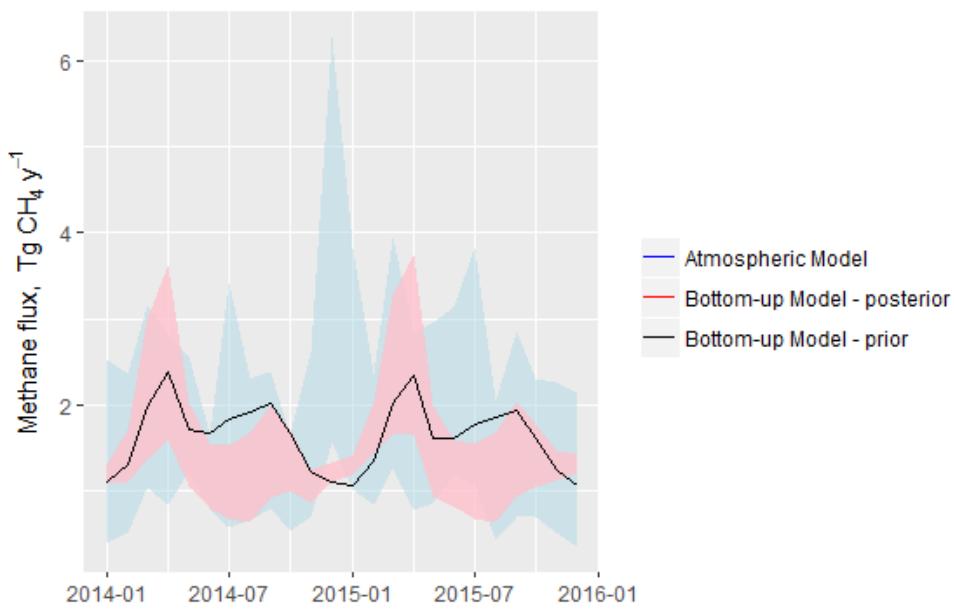


*RMS - ratio*



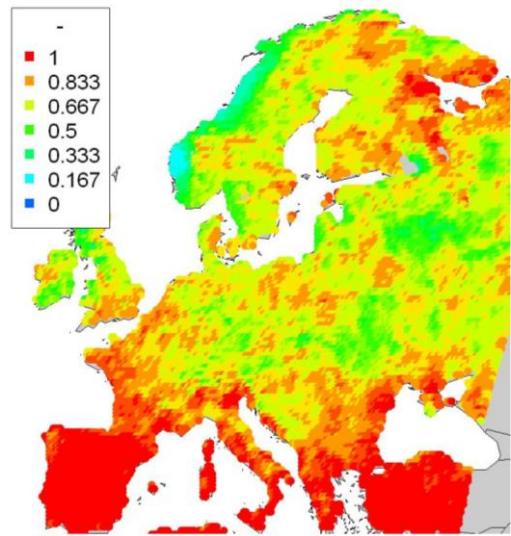
## 4.2 Reconciling LUC-inventory models with atmospheric inversion

- Each emission sector has a characteristic spatial & temporal pattern
- Use this in a Bayesian analysis of the parameters of a “bottom-up” model (anthropogenic & biogenic)
- Derive sector-wise scaling factors = multipliers needed to match atmospheric model
- Posterior distribution for methane shown (2014-2015)

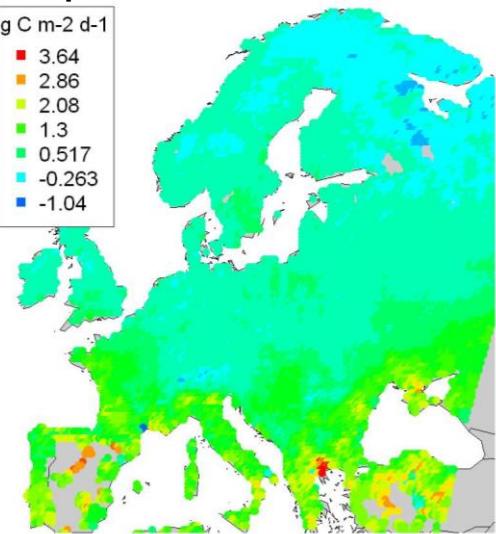


## 4.5 Drought-risk analysis for NPP in 2071-2100

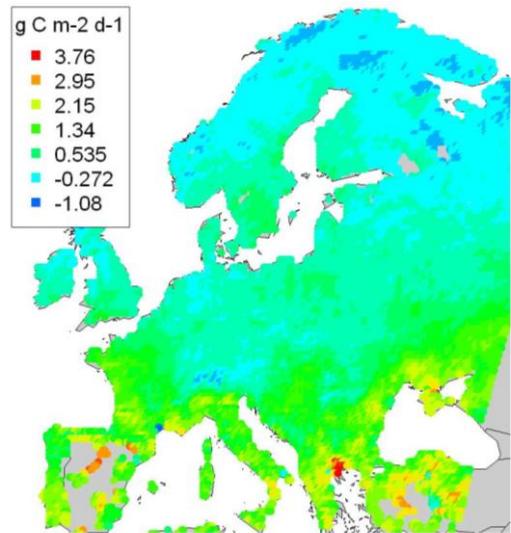
Probability of drought (PH)



Risk (R)  
= expectation of loss



Vulnerability (V)



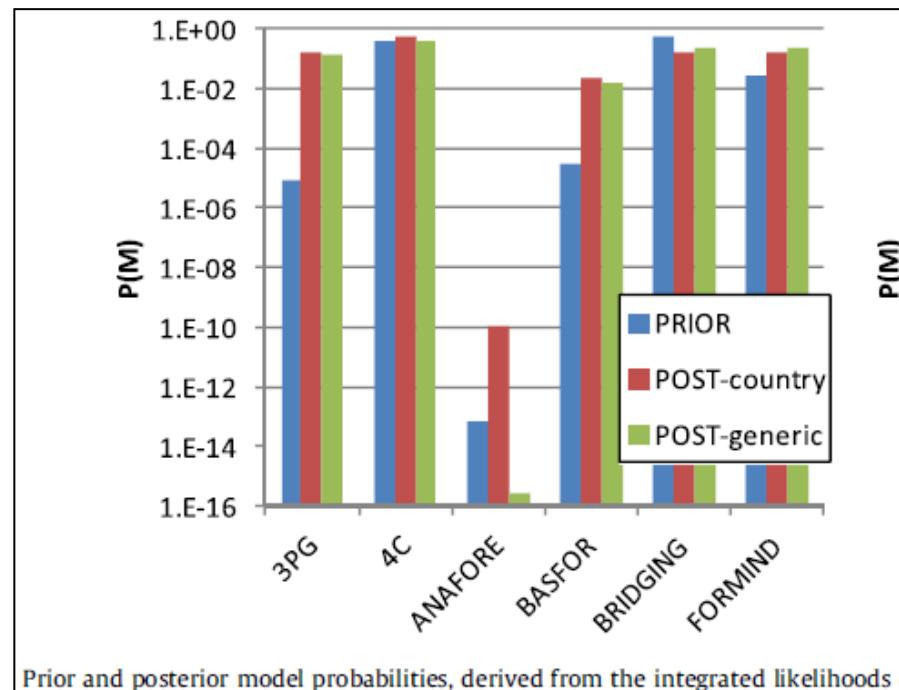
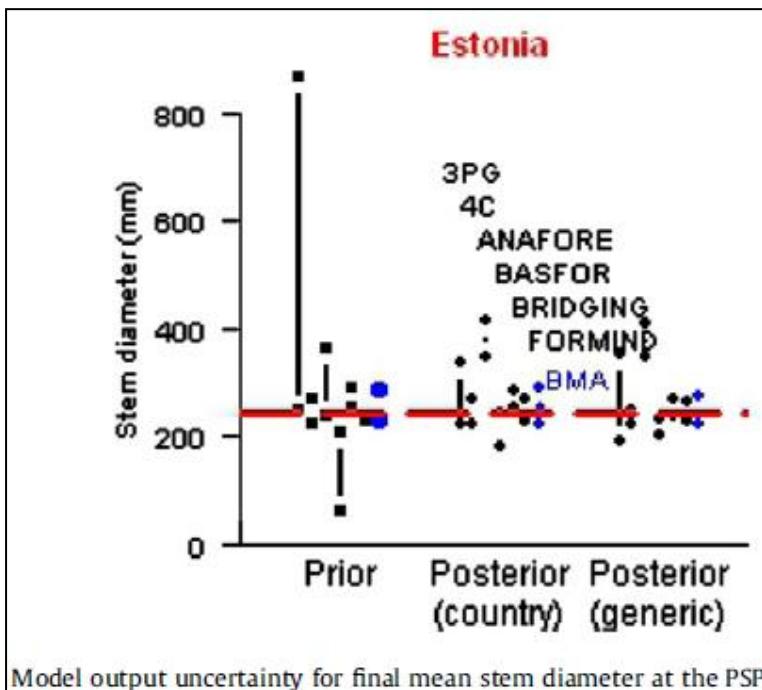
$$R = P(H) V$$

### 3.8 Bayesian calibration of process-based models

Forest Ecology and Management 289 (2013) 255–268

Bayesian calibration, comparison and averaging of six forest models, using data from Scots pine stands across Europe

M. van Oijen <sup>a,\*</sup>, C. Reyer <sup>b</sup>, F.J. Bohn <sup>c</sup>, D.R. Cameron <sup>a</sup>, G. Deckmyn <sup>d</sup>, M. Flechsig <sup>b</sup>, S. Häkkinen <sup>e</sup>, F. Hartig <sup>c</sup>, A. Huth <sup>c</sup>, A. Kivistö <sup>f</sup>, P. Lasch <sup>b</sup>, A. Mäkelä <sup>g</sup>, T. Mette <sup>h</sup>, F. Minunno <sup>i</sup>, W. Rammer <sup>j</sup>



# INTERMEZZO

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**Shiny app for BELUC**

**Shiny app for MU-MAP**